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CHAPTER I

INTRODUCTION

The Problem

The cotton producers in the United States lost an estimated $125,000,000 \(^1\) in 1951 as a result of price weaknesses in the cotton market. The blame for this price weakness was directed at the Crop Reporting Board's forecasts in the summer of 1951 of a very large cotton crop which failed to materialize. This and scattered rumblings of dissatisfaction over various reports and estimates made by the Crop Reporting Board led to a study of the Board's methods and procedures. This study was conducted by a special subcommittee of the Committee on Agriculture of the House of Representatives. The special subcommittee made the following general statement. \(^2\)

"The committee is of the opinion that the usefulness and accuracy of crop estimates would be improved if it were possible for the board to use more objective data against which the opinions and subjective data obtained from its crop reporters could be balanced. By 'objective data' the committee means the type of information which is obtained from cotton boll counts, field measurements of corn and other grains, use of crop meters, moisture tests and similar observations where definitely factual data is obtained."

The early cotton production forecasts are based on an estimated acreage times yield per acre expansion. Any error in measuring either factor can result in errors of production forecasts. This study will be concerned with errors in acreage estimates and will compare four methods of measuring acreage in cotton fields. The four methods of measurement considered are:

i. Surveyors' chain measurements.

ii. Farmers' estimates.

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\(^1\) Report and Recommendations of a special subcommittee of the House of Representatives, Eighty-second Congress; Second Session, p. 10.

\(^2\) Ibid., p. 28.
iii. Planimeter measurements of aerial photographs.

iv. Rotometer measurements of aerial photographs as made by the Agricultural Stabilization and Conservation Service of USDA (ASC).

The 1951 Cotton Estimates

The publication of "relatively unbiased" statistics of production and consumption is a necessity for the smooth functioning of supply and demand in our present society. The producer, handler, processor and government must have an up-to-date idea of actual supply if they are to perform their role in this complex modern economy. If these statistics do not accurately reflect the picture someone from the initial producer to the retailer may be subject to large financial losses. Prices to the consumer also may be a reflection of the error.

The 1951 cotton production estimates illustrate this point. The estimate of 29,510,000 acres under cultivation as of July 1, 1951, published on July 9, 1951, was the highest cotton acreage estimate published for that date since 1937. Following this indication of a larger-than-usual crop, the price of cotton fell. In early July before the report was released, the average price of middling 15/16-inch cotton had been 44.79 cents per pound. After the publication of the report the price fell until by the end of July it was at 36.21 cents per pound. The price continued to drop slowly following the publication of the first production forecast in August. The price remained relatively stationary until mid-September when outside forecasts began to throw some doubt on the Board's high production forecasts. The price climbed gradually as later reports and forecasts reduced the high original indications and forecasts. The estimate of 29,510,000 acres under cultivation at July 1 was later finalized at 27,917,000 acres, or a reduction of 5.4 percent from the original acreage estimates.

The original acreage estimates are subject to reduction due to abandonment, much of which occurs after July 1. This is not reflected in the estimate of
acres made as of July 1. The August estimate is corrected for abandonment on the basis of the 10-year average natural abandonment. It is not until the September estimate that current year abandonment figures are taken into account in the estimate. The July acreage estimate serves as a base for all succeeding monthly estimates of acreage which correct the July estimate for abandonment until the final acreage estimate is released in December. These monthly estimates of acreage from July through September are fundamental in all production forecasts. The original acreage estimates even play an important role in late season forecasts.

The first forecast of production is made as of August 1. These figures are obtained by multiplying the basic acreage estimate for each area, by the August 1 indication of average yield per acre. The September production forecast is obtained by multiplying the estimate of September acreage by the forecast of yield per acre as of September 1. Thus the July acreage estimate is fundamental in forecasting the cotton crop in the succeeding two months. Any error in the basic acreage will be reflected in the forecast.

The July 1 acreage figure is obtained from the July 1 reports of a group of special cotton correspondents in each state, who report on cotton acreage on their farms. This acreage is corrected for errors and bias by the use of regression charts. These regression charts are graphic charts showing a linear, historic relationship between the sample indications and the actual acreage at that date. The charts are designed to remove any subjective bias in the acreage figure supplied by the correspondent.

A considerable amount of subjective judgement is inherent in the making and use of these regression charts. The basic regression line is drawn freehand since the usual mathematical fitting gives undue emphasis to extreme values. The precision of the readings of the corrected estimates read on the regression line against the farmers' estimate of acreage depends on the subjective precision of the person doing the reading.
A second subjective assumption is the abandonment deduction used for the August 1 acreage estimate. This assumes that the average acreage abandonment for the past 10 years is the abandonment which has occurred currently. The July 1 acreage estimate less the average abandonment figure is used as the acreage base for the yield per acre expansion in the first cotton production forecast. Apparently this figure for abandonment is used regardless of the known conditions of weather and abandonment.

With this amount of subjective judgement entering into the making of cotton acreage estimates and forecasts, it appears that more objective data could be used.
Cochran (1953) states,

"The problem of errors of measurement is an old one in physics, chemistry and biology. A large amount of information has been gathered about the behavior both of instruments of measurement and of the human observer who plays a role in many measuring techniques. Much of this knowledge should be applicable to errors of measurement which occur in sample surveys."

The procedures for measuring errors in the applied sciences come under two general headings.

i. Comparison of results in respect to a standard (a true value)

ii. Comparison between results of two or more determinations using the same or different methods.

The first procedure has led to the establishment of national testing laboratories whose function is to compare results of experiments with certain basic well-defined standards. This method of testing for errors by the use of standards presents some difficulties in sampling work. One problem is, what is a standard for the item sampled?

Hansen, Hurwitz, Marks and Maudlin (1951) have advanced some criteria for a true value.

i. The true value must be uniquely definable.

ii. The true value must be defined in such a way that the purpose of the survey is met. That is, it must be the correct answer to answer the inquiry.

iii. The true value must be defined in terms of operations which can be carried through.

It is usually possible to define the variable under study in such a manner as to satisfy two of the three criteria. Satisfying all three is more difficult. This is particularly so when the response is verbal to a verbal question, as in an interview, which refers to a person's opinion or preference. It is virtually
impossible to define a true "brand preference" or a true "opinion". An opinion can never be defined as can the physics definition for one horsepower.

Even when it is possible to fully specify the information desired there may be difficulty obtaining the desired information because of the correspondent's or interviewee's lack of knowledge or cooperation.

Comparison between results is the most common information sought in applied science experiments. Here the interest lies in the difference between various measurement schemes and an accurate standard in order to determine whether the measurements are unbiased in respect to one another.

Mahalanobis (1946) discusses procedures used in survey work in India to check for, and to evaluate where possible, errors in sampling. One of the procedures used was to divide the sample for a given area into two or more independent, but interpenetrating and random networks of sample units each of which covers the same area. These have been called interpenetrating subsamples. By assigning the interpenetrating sample units to different investigators it is possible to detect differences in results. In some cases the same sample is investigated by more than one team of investigators.

A discussion of this technique in sampling with an example carried out in Baltimore was presented by Hansen, Hurwitz et al (1951). The sampling aspect investigated was the contribution of interviewer error to the over-all error. Cost factors of different procedures were considered. In the same paper a double sampling scheme to correct for response bias was presented. This entailed a relatively inexpensive but biased primary sample and an accurate subsample. From a comparison of the accurate subsample and the corresponding portion of the main sample, a correction factor may be found to correct for the bias in the main sample and provide a more accurate estimate.
Procedures used by the Census Bureau to reduce response errors in census returns were discussed in a paper by Marks and Mauldin (1950). This was by the use of pretests to check on the response to various types of schedules before the main survey or census.

Durbin and Stewart (1951) reported on an investigation of interviewer performance conducted by the Division of Research Techniques, London School of Economics. Booker and David (1952) added further comments to the investigation. The problem investigated was to study the differences between trained and untrained interviewers.  

In this study a systematic sample was drawn from the National Register for each of three London boroughs. Three groups of interviewer teams were drawn from three sources, one group from volunteer students of the school and two groups—one from each of two professional interview organizations. Three different interview schedules were used making nine combinations of interviewer group and questionnaire. The combinations were applied, in order, to the listing of each sample, and the sample separated into its nine parts for assignment among the three groups of teams. For practical reasons each team worked in one district only rather than in all three, but did use each of the three different schedules.

The data collected were analyzed as a factorial experiment. Results showed there was no clear evidence for assuming that differences in results arose from differing abilities or that the inexperience of the amateur interviewers led to recording opinion, preferences or facts significantly different from those recorded by experienced interviewers. The quality of the professional interviewer's work was somewhat superior to that of the amateur interviewers.

1/ This question is of great importance to survey organizations who are faced with the question of recruiting field workers and wish to know how much of their resources can be or should be devoted to the training of interviewers.
In a study made by Kish and Lansing (1954), the results of an owner's appraisal of his home was contrasted against the valuation placed on the home by a professional appraiser. A subsample of the homes reported on in the 1950 Survey of Consumer Finances was visited by professional residential appraisers who placed what they considered a correct value on the home. The two values were paired and put into specified groupings of valuation; owner's income, age, education, etc. for purposes of analysis and testing of certain specified hypotheses.

The model used to investigate the data was the expansion of \( E(r_i - \bar{R})^2 \), expanded in two ways and the results equated. The resulting relationship was

\[
V(r) + \bar{d}^2 = V(a) + M.S.(d) + 2Cov.(d,a)
\]

where

\( r_i \) = owner's appraisal of \( i^{th} \) home with mean \( \bar{R} \).

\( a_i \) = appraiser's appraisal of \( i^{th} \) home with mean \( \bar{A} \).

\( d_i \) = difference between \( r_i \) and \( a_i \) with mean \( \bar{d} \).

The values of \( M.S.(d) \); the mean square difference of the measurements; and \( \bar{d}^2 \), the unbiased estimate of the net bias squared, were examined in relationship to the variance of the home owners' and appraisers' valuations. In spite of a large \( M.S.(d) \) value the net bias \( \sqrt{\bar{d}^2} \) was quite small, as most of the discrepancies cancelled, leaving a small net average error. The article drew no definite conclusions as to the significance of the net bias values found under the several hypotheses made.
CHAPTER III

THE DATA

A. Sampling Design

Sampling units (SU) of cotton fields were drawn from three counties: Cleveland, Iredell and Union, in the Southern Piedmont area of North Carolina. Cotton is the predominant crop in this area. Area sampling was used, and an attempt was made to get clusters of four cotton fields per SU.

In order to accomplish this, information was obtained from the North Carolina State Office of the Agricultural Stabilization and Conservation Service (ASC) and the Agricultural Extension offices at North Carolina State College on the number of cotton fields per farm. From this information the number of cotton fields per county was estimated. The estimated number of SU in each county was obtained by dividing the estimate of the number of cotton fields per county by four.

On maps of each county showing the most recent culture, area segments were designated with clearly defined boundaries, (roads, rivers and well-defined streams). Each of these area segments was defined as count units (CU). The indicated number of farms (INOF) was counted for each CU, and the INOF was cumulated for the county. The INOF count for each county was then related to the number of SU assigned, in order to determine the approximate number of SU per CU.

A sample of 100 cotton fields was desired; hence 25 SU, each with an expected four cotton fields, were selected at random with equal probability. Nine SU's were selected from Cleveland and eight SU's from each of the other two counties. To select the SU's in a county, for example, Cleveland County, nine random numbers between one and the total number of SU's in Cleveland County were drawn. Each random number indicated a particular SU in a particular CU, according to the cumulative INOF listing table. The indicated CU was allotted a number of SU's to the
nearest integer by dividing the INOF in the CU by the ratio of the county INOF total to the county SU total. To reduce the subsampling wherever possible the CU's were further sub-divided, keeping well-defined boundaries. In many cases it was necessary to subsample the entire CU to obtain the SU selected.

B. The Measurements

i. Chain measurements

Each farmer who cultivated at least one field of cotton in the CU selected had his cotton field or fields listed on a screening schedule. From this list of fields, the cotton fields in the SU were selected and permission obtained from the farm operators to measure the sample fields. Chain measurements were then made in the sample fields by one of three teams, each consisting of two men.

Of the 25 SU's originally selected, 22 were screened. These 22 SU's contained 60 cotton fields which constituted the sample. When the screening schedule information for the 22 SU's was examined it was found that the basic data used in determining the sampling rates had been in error. The number of fields had been overestimated, and the acreage per field had been underestimated. While the average field in the sample contained four acres instead of three, there were 30.7 percent less fields than expected. Total sample acreage was 7.6 percent less than expected. Only in Iredell County did the actual number of fields exceed the expected number.

Work was done in this county last, and only five of the eight SU's were measured. Table 1 gives a summary of the expected and actual screening schedule information.

As it had taken longer to measure the fields than was anticipated, measurements had to be abandoned in order to gather other objective field data for the August 1 production forecast. Also that portion of the budget allotted for measuring fields had been expended after 60 fields were measured.

Several fields selected at random from the sample fields in each county were remeasured at a later date. In Iredell County, 7 fields originally measured by
team B were remeasured by Team A. Team A remeasured 7 fields in Union County originally measured by Team C. In Cleveland County, 5 fields originally measured by team A were remeasured by team B; then team A again measured the same 5 fields. In all, there were two measurements on 14 fields and three measurements on 5 fields.

Table 1. Screening summary.

<table>
<thead>
<tr>
<th>County</th>
<th>SU's in Sample</th>
<th>Fields</th>
<th>Acreage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
<td>Expected</td>
</tr>
<tr>
<td>Cleveland</td>
<td>9</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>Union</td>
<td>8</td>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>Iredell</td>
<td>8</td>
<td>22</td>
<td>(20)</td>
</tr>
</tbody>
</table>

1/ Only five out of eight SU's measured.

Chain measurements were considered the most accurate and objective of the four measurement schemes. This method of measuring was regarded as an unbiased technique which was subject only to a random measuring error. It was considered as the basis for comparing the other measuring techniques.

ii. Farmers' Estimates

At the time of the screening of the CU each farmer or farm operator in the CU was asked to estimate the acreage in each of his cotton fields. This estimate was later used for comparison with the chain measurement.

The farmers' estimates were considered to contain errors of two types; a systematic error and a random error. Where the field had been measured on a previous occasion, probably little of the systematic error (an individual bias) existed. Where the farmer had only a good guess as to the true value, he might consistently underestimate or overestimate the area by a substantial amount and hence report with a larger bias. The presence of an acreage allotment on cotton forced
the farmer to keep a close check on his acreage planted. Therefore the reporting bias is not expected to be large. The individual reporting bias might, on the other hand, be considerable for crops not under fixed acreage allotment. In connection with the farmers' reporting it should be noted that the rounding errors in the reporting are greater than for the other methods of measuring -- since these reports are to the nearest acre only in many cases. A few farmers reported to the nearest tenth of an acre.

iii. Planimeter

Aerial photographs were carried by the enumerator on his second visit to the sample fields, and boundaries of the fields were delineated on these photographs. In the Raleigh office, each field was measured independently by three clerks using a planimeter in order to estimate the acreage in the field.

The amount of recording error for this type of measurement depends on the skill of the operator. Nevertheless, it should be small. A source of much larger error is the subjective decisions introduced in delineating the field on the aerial photograph.

iv. ASC Measurements

Acreage data for the fourth type of measurement were obtained from the ASC office in each county. These offices were visited and the sample fields were located on their aerial photographs wherever possible. Their rotometer measurements of the same fields were taken to compare with the other acreage data. The acreage figures obtained from the county ASC offices were basically rotometer readings but other methods might be employed if the farmer requested a remeasurement. The remainder of this paper will refer to these acreage figures as ASC measurements.
Four methods of measurement were used to obtain the data used in this study. The measuring teams were supervised closely to insure that instructions were followed closely and uniformly. Each of the methods of measurement indicated definite characteristics when contrasted to chain measurements. These characteristics will be discussed in a later chapter.
CHAPTER IV

METHOD OF ANALYSIS

The Models

The populations which can be generated from the various methods of measuring acreage have two parameters that are of interest, the means and the variances. The difference between the means of the three methods and the mean for the chain measurement will indicate the amount of bias present. The variances will give a basis for comparing the efficiencies of the various methods. Further analysis of the components of variance give valuable information for comparing methods and for planning any future work in the field.

The measure of variability of a biased estimate is called the mean square error (MSE) and usually has two components as shown in (1).

\[ \text{MSE} = \sigma^2 + B^2 \]  

(1)

where

\( \sigma^2 \) = variance (around the expected value)

and

\( B^2 \) = square of the net bias.

The relative size of the variance and bias squared depends on the accuracy and precision with which each observation in the sample is made. Under optimum conditions where the true values of a sample can be measured without bias, the MSE of the mean with random sampling reduces to \( \sigma^2/n \) (ignoring the finite population correction factor). With a biased estimate it is ordinarily worthwhile to examine the composition of the MSE if possible.

Consider the composition of a simple randomly selected observation taken from a population with mean \( \mu \). If this observation is obtained with an unbiased measuring device, then the \( j^{\text{th}} \) measurement in the \( i^{\text{th}} \) unit in the sample \( (y_{ij}) \) might be represented by Model I.
\[ y_{ij} = u + t_i + \delta_{ij} \]  
(2)

where

- \( t_i \) = the difference between the true value of the \( i^{th} \) unit and the population mean. The \( t_i \)'s have mean zero and variance \( \sigma_t^2 \) and
- \( \delta_{ij} \) = the error associated with the \( j^{th} \) measurement of the \( i^{th} \) unit measured in the sample. The \( \delta_{ij} \)'s are independently distributed with mean zero and variance \( \sigma_\delta^2 \). The \( \delta \)'s and \( t \)'s are assumed to be uncorrelated.

The variance of a single unbiased observation is

\[ \sigma_y^2 = \sigma_t^2 + \sigma_\delta^2 \]  
(3)

If the measurement scheme adopted is biased, then the observation on the same \( i^{th} \) unit may be represented by Model II

\[ x_{ij} = u + t_i + b_i + \varepsilon_{ij} \]  
(4)

where

- \( b_i \) = the individual bias associated with the measurement of the \( i^{th} \) unit in the sample, the mean value of \( b_i \) is \( B \), with variance \( \sigma_b^2 \).
- \( B \) = the expected or net bias for this measuring scheme. The \( b_i \)'s may be correlated with the \( t_i \)'s.
- \( \varepsilon_{ij} \) = the error associated with the measuring of the \( j^{th} \) observation of the \( i^{th} \) unit under the same sampling scheme. It has mean zero, variance \( \sigma_\varepsilon^2 \) and is assumed to be uncorrelated with the \( b_i \)'s and \( t_i \)'s.

The variance for a single observation with Model II is

\[ \text{Var} (x_{ij}) = \sigma_t^2 + \sigma_b^2 + \sigma_\varepsilon^2 + 2\rho_{bt} \sigma_b \sigma_t. \]  
(5)
Proof:

\[ \text{Var} (x_{ij}) = E (\bar{x}_{ij} - E(x_{ij}))^2 \]
\[ = E \left( \bar{t}_1 + t_i + b_i + e_{ij} - (u + B) \right)^2 \]
\[ = E \left( \bar{t}_1 + (b_i - B) + e_{ij} \right)^2 \]
\[ = E(t_1)^2 + E(b_i - B)^2 + E(e_{ij})^2 \]
\[ + 2E(t_1)(b_i - B) + 2E(t_1)(e_{ij}) \]
\[ + 2E(b_i - B)(e_{ij}) \]
\[ = \sigma_t^2 + \sigma_b^2 + \sigma_e^2 + 2\rho_{bt} \sigma_b \sigma_t \cdot \]

The \( e_{ij} \)s are uncorrelated with the other components so cross product terms involving \( e_{ij} \) become zero.

The variance of the mean is
\[ \sigma_{\bar{x}}^2 = \sigma_x^2 / n = \frac{1}{n} [\sigma_t^2 + \sigma_b^2 + \sigma_e^2 + 2\rho_{bt} \sigma_b \sigma_t] \quad (6) \]

The other component of \( \text{MSE}(x) \) is \( B^2 \). \( B \) is the difference between the population mean \( u \) and expected value of the \( x_{ij} \)s, i.e.
\[ B = (E(x_{ij}) - u). \quad (7) \]

If a random sample of \( n \) elements is selected and two measurements made on each sample element, one biased and the other unbiased, then the best sample estimate of \( B \) is the difference between the means; i.e.,
\[ B = (\bar{x} - \bar{y}) \quad (8) \]

However, \( (\bar{x} - \bar{y})^2 \), the apparent estimate of \( B^2 \) is a consistent but not unbiased estimate of \( B^2 \). The expectation of the square of the difference between two such sample means is given in the following relationship.
\[ B^2 = E(\bar{x} - \bar{y})^2 = \sigma^2(\bar{x} - \bar{y}) \quad (9) \]

\[ \rho_{bt} \sigma_b \sigma_t \] is identical with \( \sigma_{bt} \), which will be used for the remainder of this paper.
Proof:

\[ E(\bar{x} - \bar{y})^2 = E(\bar{x} - E(\bar{x}) - \bar{y} + E(\bar{y}) + E(\bar{x}) - E(\bar{y}))^2 \]
\[ = E(\bar{x} - E(\bar{x}))^2 + E(\bar{y} - E(\bar{y}))^2 + E(E(\bar{x}) - E(\bar{y}))^2 \]
\[ - 2E(\bar{x} - E(\bar{x}))(\bar{y} - E(\bar{y})) \]
\[ + 2E(\bar{x} - E(\bar{x}))(E(\bar{x}) - (\bar{y})) \]
\[ - 2E(\bar{y} - E(\bar{y}))(E(\bar{x}) - E(\bar{y})) \]
\[ = \sigma_x^2 + \sigma_y^2 - 2\sigma_{\bar{x} \bar{y}} + \beta^2, \]

since

\[ (E(\bar{x}) - E(\bar{y})) = \beta. \]

Thus

\[ \beta^2 = E(\bar{x} - \bar{y})^2 - \sigma_{\bar{x} \bar{y}}. \]

The variance equation of Model II (5) contains \( \sigma_t^2, \sigma_b^2, \sigma_\varepsilon^2 \) and a covariance term. Estimates of \( \sigma_t^2 + \sigma_\varepsilon^2 \) may be obtained by methods that will be explained later. The covariance between two methods of measurement, under our assumption, is

\[ \sigma_{xy} = \sigma_t^2 + \sigma_{bt}. \quad (10) \]

Proof:

\[ \sigma_{xy} = \text{Cov} (x_{ij}, y_{ij}) \]
\[ = E(x_{ij} - E(x_{ij}))((y_{ij} - E(y_{ij})) \]
\[ = E(u + t_i + b_i + \varepsilon_{ij} - (u+B)\gamma/u + t_i + \delta_{ij} - u)^2 \]
\[ = E(t_i^2 + (b_i - B) + \varepsilon_{ij} + \delta_{ij}) \]
\[ = E(t_i^2) + E(t_i)(\delta_{ij}) + E(b_i - B)(t_i) \]
\[ + E(\varepsilon_{ij})(\delta_{ij}) \]
\[ = \sigma_t^2 + \sigma_{bt} = \sigma_{xy} \]

as the errors \( \varepsilon_{ij} \) and \( \delta_{ij} \) are independently distributed and \( E(\varepsilon_{ij}) = E(\delta_{ij}) = 0 \).
The results of the field work using the unbiased measuring scheme gave \( n \) measurements on \( m \) fields. Not all the fields were measured the same number of times. If the results are analyzed using the analysis of variance technique for unequal subclass numbers, they may be summarized as follows:

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>D.F.</th>
<th>SS</th>
<th>MS</th>
<th>E(ES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between fields</td>
<td>( m-1 )</td>
<td>SSF</td>
<td>MSF</td>
<td>( \sigma^2_0 + n_o \sigma^2_t )</td>
</tr>
<tr>
<td>Between measurements</td>
<td>( n-m )</td>
<td>SSE</td>
<td>MSE</td>
<td>( \sigma^2_0 )</td>
</tr>
<tr>
<td>Total</td>
<td>( n-1 )</td>
<td>SST</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this particular study, the between measurement variance, \( \sigma^2_0 \), is made up of two parts:

1. \( \sigma^2_{mt} \) = the variance component for repeated measurements by different measuring teams on the same field.
2. \( \sigma^2_\gamma \) = the variance component for repeated measurements by the same team on a given field.

Both of these variance components are assumed to remain the same for all fields; in addition \( \sigma^2_\gamma \) is assumed to be constant for measuring teams. The data available for all three counties were analyzed to obtain estimates of these components. The analysis yielded \( \hat{\sigma}^2_{mt} = .031 \) and \( \hat{\sigma}^2_\gamma = .025 \). The number of remeasurements made by the same and different teams was insufficient to break out these components for each county. However, \( \hat{\sigma}^2_\gamma \) was very small relative to the variation among the fields (\( \sigma^2_t \)). Hence a further breakdown of this component was unnecessary.

Only one measurement per sample unit was available for the biased methods of measurement. Thus it was not possible to obtain a separate estimate of \( \sigma^2_e \). Assuming no error in field delineation, \( \hat{\sigma}^2_e \) could have been obtained for planimeter measurements since three separate readings were made for each field. From other studies
errors of delineation, however, appear to be of greater magnitude, so \( \sigma^2_e \) obtained from triplicate readings might be considerably underestimated. It appears impractical to repeatedly ask a farmer the acreage in a particular field although repeated questions might be possible over time. Theoretically one would assume that the farmer's answers would be constant and present a bias from the true acreage rather than a random fluctuation around the true acreage. This does not detract from the values of the model, except that, with only one answer from the farmer, it is not possible to separate measuring error from the variance of the bias (i.e., \( \sigma^2_b \) from \( \sigma^2_e \)).

It will be noted that for some fields there are two or three unbiased measurements for each biased measurement. Thus to obtain the sum of products for computing covariance it was necessary to repeat the biased measurement whenever there was more than one unbiased measurement. The interpretation of the resulting sum of products for the covariance term is not straightforward as in the case where single \( x \)'s and \( y \)'s may be paired. An examination of the analysis of variance of the products of \( x_i \) and \( y_{ij} \) may be helpful in the interpretation of the covariance.

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>D.F.</th>
<th>SP</th>
<th>MSP</th>
<th>E(MSP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between fields</td>
<td>m-1</td>
<td>SPB</td>
<td>SPB</td>
<td>( \sigma^2_z + K_o \sigma_{xy} )</td>
</tr>
<tr>
<td>Within fields</td>
<td>n-m</td>
<td>SPW</td>
<td>SPW</td>
<td>( \sigma^2_z )</td>
</tr>
<tr>
<td>Total</td>
<td>n-1</td>
<td>SPT</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Referring to Model I and Model II and the assumptions, it will be noted that the error terms \( \varepsilon_{ij} \) and \( \delta_{ij} \) are independent, so uncorrelated. Therefore the within sum of products for error will be zero and \( \sigma^2_z = 0 \). \( \hat{\sigma}_{xy} \) can now be obtained by equating MSP to E(MSP) and solving; i.e.,

\[
\frac{BSP}{m-1} = K_o \sigma_{xy} \\
= \frac{1}{m-1} \sum_{k_i} k_i - \frac{\sum (k_i)^2}{\sum_{k_i}} \hat{\sigma}_{xy}
\]
\[ \hat{\sigma}_{xy} = \frac{BSP}{\sum k_i - \frac{\sum (k_i)^2}{\sum k_i}} \] (11)

where

\[ k_i = \text{number of chain measurements on the } i^{th} \text{ field in the sample.} \]

In summary, the various components of the MSE can be estimated as follows:

i. \((\bar{x} - \bar{y})\) estimates \(B\).

ii. \(S_x^2\) estimates \(\sigma_t^2 + \sigma_b^2 + 2\sigma(bt) + \sigma^2\).

iii. \(S_{xy}\) estimates \(\sigma_t^2 + \sigma(bt)\).

iv. \((\bar{x} - \bar{y})^2 = \frac{S^2(x-y)}{n}\) estimates \(B^2\).

v. Results of the analysis of variance estimate \(\sigma_t^2\) and \(\sigma_b^2\).

It should be remembered that with these data, however, \(\sigma_t^2\) cannot be separated from \(\sigma_b^2\).
CHAPTER V
RESULTS AND DISCUSSION

The acreage data, obtained by using each of the four measuring schemes on the sample of cotton fields, was analyzed using the techniques developed in Chapter IV. A summary is given in Table 2. It is given for each county and for the combined total. The third column, the difference between the means, is the net difference between measuring schemes and an estimate of B, the net bias. The remaining columns give the components of the MSE of the measurement.

In examining the mean acres per cotton field, the farmers' estimate was found to be almost the same as the mean for chain measurements, which were considered unbiased. This would seem to indicate that if any individual bias is present, it cancels out. The farmers' estimate of acreage in Iredell County was an underestimate of 7 percent. In Cleveland and Union counties the farmers' estimate overestimated but by less than 3 and 1 percent respectively. The planimeter measurements overestimated in each county. The overestimate was only 3 percent in Iredell County but was about 10 percent in Cleveland and about 8 percent in Union. The ASC measurements underestimate the mean acreage per cotton field for chain measurements in all counties. The difference varies from a little more than 2 percent in Iredell to 5 percent in Union and 11 percent in Cleveland.

For the combined totals of the three counties, the farmers' estimate was low by 1 percent, planimeter measurements overestimated by 7 percent, while ASC underestimated by 6 percent.

In view of the variances, perhaps the three counties should not have been combined. Certainly Iredell County does not present the same variance picture as the other two counties. Since the number of fields in the sample per county was small, it was considered that pooling would present a better picture. It should be noted also that the calculation of sample variances was based on an assumption of
Table 2. Summary of estimated means, differences and variances for four measuring schemes.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>Mean</th>
<th>Bias</th>
<th>MSE</th>
<th>$\hat{\sigma}^2$</th>
<th>$\hat{\sigma}_{t}^2$</th>
<th>$\hat{\sigma}_{b}^2$</th>
<th>$\hat{\sigma}<em>{b}^2 + \hat{\sigma}</em>{t}^2$</th>
<th>$2\hat{\sigma}_{b}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cleveland County</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain</td>
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<td>-</td>
<td>12.11</td>
<td>-</td>
<td>12.11</td>
<td>12.00</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>Farmers'</td>
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<td>4.61</td>
<td>0.14</td>
<td>13.20</td>
<td>-0.097</td>
<td>13.20</td>
<td>12.00</td>
<td>-1.75</td>
<td>-0.55</td>
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<td>Planimeter</td>
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<td>0.48</td>
<td>15.34</td>
<td>0.166</td>
<td>15.37</td>
<td>12.00</td>
<td>-0.94</td>
<td>2.43</td>
</tr>
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<td>5.14</td>
<td>-</td>
<td>12.11</td>
<td>-</td>
<td>12.11</td>
<td>12.00</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
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<td>11.57</td>
<td>12.00</td>
<td>-0.79</td>
<td>-1.22</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain</td>
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<td>4.20</td>
<td>-</td>
<td>11.83</td>
<td>-</td>
<td>11.83</td>
<td>11.77</td>
<td>0.06</td>
<td>-</td>
</tr>
<tr>
<td>Farmers'</td>
<td>22</td>
<td>4.21</td>
<td>0.01</td>
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<td>11.21</td>
<td>11.77</td>
<td>-2.70</td>
<td>-0.26</td>
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<td>0.38</td>
<td>15.99</td>
<td>0.034</td>
<td>15.96</td>
<td>11.77</td>
<td>-2.38</td>
<td>1.81</td>
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<td>-</td>
<td>11.83</td>
<td>-</td>
<td>11.83</td>
<td>11.77</td>
<td>0.06</td>
<td>-</td>
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<tr>
<td>ASC</td>
<td>19</td>
<td>4.23</td>
<td>-0.19</td>
<td>13.28</td>
<td>0.021</td>
<td>13.26</td>
<td>11.77</td>
<td>-0.23</td>
<td>1.26</td>
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<td>-</td>
<td>2.85</td>
<td>2.84</td>
<td>0.01</td>
<td>-</td>
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<td>Farmers'</td>
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<td>0.022</td>
<td>2.86</td>
<td>2.84</td>
<td>-0.59</td>
<td>-0.57</td>
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<td>0.08</td>
<td>3.50</td>
<td>0.001</td>
<td>3.50</td>
<td>2.84</td>
<td>-0.12</td>
<td>0.51</td>
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<tr>
<td>Chain</td>
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<td>3.35</td>
<td>-</td>
<td>2.85</td>
<td>-</td>
<td>2.85</td>
<td>2.84</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>ASC</td>
<td>20</td>
<td>3.26</td>
<td>-0.08</td>
<td>3.03</td>
<td>0.004</td>
<td>3.03</td>
<td>2.84</td>
<td>-0.07</td>
<td>0.26</td>
</tr>
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<td>3 Counties</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain</td>
<td>60</td>
<td>3.97</td>
<td>-</td>
<td>8.80</td>
<td>-</td>
<td>8.80</td>
<td>8.73</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>Farmers'</td>
<td>60</td>
<td>3.93</td>
<td>-0.04</td>
<td>9.82</td>
<td>-0.023</td>
<td>9.82</td>
<td>8.73</td>
<td>-1.41</td>
<td>-0.32</td>
</tr>
<tr>
<td>Planimeter</td>
<td>60</td>
<td>4.26</td>
<td>0.30</td>
<td>11.30</td>
<td>0.055</td>
<td>11.24</td>
<td>8.73</td>
<td>-0.89</td>
<td>1.62</td>
</tr>
<tr>
<td>Chain</td>
<td>52</td>
<td>4.19</td>
<td>-</td>
<td>8.80</td>
<td>-</td>
<td>8.80</td>
<td>8.73</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>ASC</td>
<td>52</td>
<td>3.95</td>
<td>-0.23</td>
<td>8.91</td>
<td>0.047</td>
<td>8.86</td>
<td>8.73</td>
<td>-0.31</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

$1^/$ $\nu(y) = \sigma_{t}^2 + \sigma_{b}^2$

$2^/$ Where $B^2$ is negative, it is assumed zero in computing MSE.
simple random sampling of fields whereas in the actual design the sample unit was an expected four fields. Variances computed for cluster sampling usually are greater than the variance for simple random sampling. Estimates of the cluster variance should be computed for planning future sample sizes. A recommendation to this effect will be made in a later chapter.

To test the significance of the net bias \( B \), a "t" test was used. The standard error of the difference between the means of two sampling schemes is \( \sigma_{(\bar{X}-\bar{Y})} \). When this quantity is examined in relation to the variance components of Model I and II, it is found that

\[
\hat{\sigma}_{(\bar{X}-\bar{Y})} = \sqrt{\frac{\sigma_b^2 + \sigma_\varepsilon^2 + \sigma_\theta^2}{n}}
\]

(12)

The results of the "t" test are given in Table 3. Significant differences were found for the planimeter and ASC measurements for the combined three counties (at 5

Table 3. Summary of "t" tests of bias terms.

<table>
<thead>
<tr>
<th></th>
<th>Farmers' Planimeter</th>
<th>ASC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>t</td>
</tr>
<tr>
<td>Cleveland</td>
<td>16</td>
<td>.411</td>
</tr>
<tr>
<td>Union</td>
<td>22</td>
<td>.044</td>
</tr>
<tr>
<td>Iredell</td>
<td>22</td>
<td>1.352</td>
</tr>
<tr>
<td>3 Counties</td>
<td>60</td>
<td>.253</td>
</tr>
</tbody>
</table>

* Five percent significance level.
** One percent significance level.
1/ Negative estimate for \( \sigma_b^2 + \sigma_\varepsilon^2 \).

\[
t = \frac{\hat{\sigma}}{\sqrt{\frac{\sigma_b^2 + \sigma_\varepsilon^2 + \sigma_\theta^2}{n}}}
\]
percent significance level). None of the "t" values for individual counties tested significant. One reason for this is because there are not enough degrees of freedom for any one county individually and the bias does not "settle down" until a larger sample size is tested.

As a test for consistent differences, a sign test was applied to the data. The farmers' estimate, planimeter and ASC results were separately subtracted from the chain measurements, and the sum of the positive and negative differences was tabulated. The proportion of the smaller sum was tested to find if it was significantly different from $p = .5$. Planimeter measurements in Cleveland and Union Counties were shown to be the only individual county measurements significantly different from chain measurements at the 1 percent significance level by this test. For the combined three counties farmers' estimates and planimeter measurements were significant at the 1 percent level. The ASC measurements were not significant either by individual county or in total.

The variance of the chain measured fields with one exception was less than the variance for any other sampling method. This is to be expected as the chain measurements were the most precise and should have the least amount of error. The one exception was the ASC measurement for Cleveland County. An examination of the estimates of the component parts of the variance of $x$ for this county reveals a relatively large negative value for $\sigma_{bt}$. On examining the values of $\rho_{bt}$ for the different sampling schemes, some consistency was noted. The values derived from the farmers' estimates for each county and for the combined total, are negative and close to zero. The same values associated with planimeter measurements are all positive. Both positive and negative values of $\hat{\rho}_{bt}$ were found for ASC measurements.

The correlation coefficient, $(\rho_{bt})$, indicates the relationship, if any, between the field-to-field variation ($t_i$) and the individual bias associated with each field
\( b_i \). A negative value indicates a tendency for the bias in the measuring scheme to be less than average for large fields and greater than average for small fields. The converse of this is indicated by a positive value. If \( \rho_{bt} \) is zero, no relationship exists.

To test if there is a significant association between the size of field and bias, \( \rho_{bt} \) must be calculated. It can be calculated from the identity

\[
\rho_{bt} = \frac{\sigma_{bt}}{\sqrt{(\sigma_b^2)(\sigma_t^2)}}
\]

(13)

and the null hypothesis \( (H_0: \rho_{bt} = 0) \) tested. In order to use this identity to calculate \( \rho_{bt} \) from the data available, an assumption was necessary since there was no separate estimate of \( \sigma_b^2 \). The assumption was made that \( \sigma_b^2 \) is approximately zero. The value for \( \rho_{bt} \) calculated under this assumption would be a lower bound for \( \rho_{bt} \).

The values calculated for \( \hat{\rho}_{bt} \) under this assumption are given in Table 4.

Table 4. Calculations of \( \hat{\rho}_{bt} \).

<table>
<thead>
<tr>
<th>County</th>
<th>Farmers' Planimeter</th>
<th>ASC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( n )</td>
<td>( \hat{\rho}_{bt} )</td>
</tr>
<tr>
<td>Cleveland</td>
<td>16</td>
<td>-.060</td>
</tr>
<tr>
<td>Union</td>
<td>22</td>
<td>-.023</td>
</tr>
<tr>
<td>Iredell</td>
<td>22</td>
<td>-.220</td>
</tr>
<tr>
<td>3 Counties</td>
<td>60</td>
<td>-.046</td>
</tr>
</tbody>
</table>

\(^A\) Significant at 10 percent probability level.

\(^B\) Significant at 20 percent probability level.

\(^1\) Negative estimate for \( \hat{\rho}_b \) + \( \hat{\rho}_e \).

\(^*\) Five percent significance level.

\[
\hat{\rho}_{bt} = \frac{\hat{\rho}_{bt} \sigma_b \sigma_t}{\sqrt{\sigma_t^2 (\sigma_b^2 + \sigma_e^2)}}
\]

Assuming \( \sigma_e^2 \approx 0 \), \( H_0: \rho_{bt} = 0 \)
The only significance at the 5 percent level noted in the table for $\hat{\rho}_{bt}$ is for the combined three counties for planimeter measurement. At the 10 percent significance level $\hat{\rho}_{bt}$ for planimeter measurements for Iredell County and ASC measurements for Union County were significant. Other values were not found significant.

The results suggest that there is some association between measurement bias and size of field for the planimeter and ASC measuring procedures. The value of $\hat{\rho}_{bt}$ for farmers' estimates is near zero. The low value could be the result of assuming that $\sigma^2$ for farmers' estimates was zero for this test, when in actual fact it may be relatively large. On the other hand there may be no association between individual bias and size of field with the farmers' estimate.

Table 5 was constructed to show the effects of varying sample size on the magnitude of MSE ($\bar{x}$). The combined three county summary given in Table 1 is used as a base. Since $B^2$ turned out to be negative for the farmers' estimates it was replaced by the net bias squared, i.e., $B^2 = (\bar{x} - \bar{y})^2 = .002$.

It will be noted that the variance of the mean can be reduced in size by increasing $n$ (the sample size) but a level is reached where the contribution to MSE ($\bar{x}$) by $B^2$ becomes predominant since $B^2$ is independent of sample size. When the bias squared becomes the dominant factor the only manner in which the MSE ($\bar{x}$) can be appreciably reduced is by introducing better and more accurate measuring techniques to reduce $B$.

**A Regression Approach**

Another method of analysis makes use of the regression technique. For this analysis, regression coefficients were computed for each of the three biased methods on the chain measurements. These results are shown in Figures 1, 2 and 3.

The difference between the 1:1 line and the estimated line is a measure of average bias for a given $X$. If the estimated regression coefficient is significantly
Table 5. Summary of Bias and Variance Relationships.

<table>
<thead>
<tr>
<th>n</th>
<th>Sampling scheme</th>
<th>MSE($\bar{x}$)</th>
<th>$\bar{\sigma}^2$</th>
<th>Var. of mean</th>
<th>$\bar{\sigma}^2$</th>
<th>Var ($\bar{x}$)</th>
<th>$\sqrt{\text{MSE}}(\bar{x})$</th>
<th>Efficiency MSE($\bar{x}$) chain</th>
<th>Efficiency MSE($\bar{x}$) other</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Chain</td>
<td>.176</td>
<td>-</td>
<td>.176</td>
<td>-</td>
<td>100.0</td>
<td>10.6</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Farmers'</td>
<td>.198</td>
<td>.002</td>
<td>.196</td>
<td>0.8</td>
<td>99.2</td>
<td>11.3</td>
<td>88.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Planimeter</td>
<td>.280</td>
<td>.055</td>
<td>.225</td>
<td>19.5</td>
<td>80.5</td>
<td>12.4</td>
<td>62.9</td>
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</tr>
<tr>
<td></td>
<td>ASC</td>
<td>.224</td>
<td>.047</td>
<td>.177</td>
<td>21.0</td>
<td>79.0</td>
<td>10.3</td>
<td>78.5</td>
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</tr>
<tr>
<td>100</td>
<td>Chain</td>
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<td>-</td>
<td>.088</td>
<td>-</td>
<td>100.0</td>
<td>7.4</td>
<td>100.0</td>
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<tr>
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<td>.002</td>
<td>.098</td>
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<td>98.4</td>
<td>8.4</td>
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<td>.112</td>
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<td>9.6</td>
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<td>.089</td>
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<td>.049</td>
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<td>86.8</td>
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<td>.056</td>
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* $B^2$ negative. Assume $B^2 = (\bar{x} - \bar{y})^2 = .002$. 
different from 1.0, the size of the bias in the measurement of a field changes with size of field.

In the regression of farmers' estimates on chain measurements, the regression coefficient of 1.0017 is not significantly different from 1.0. This indicates that any bias present was too small to be detected.

The regression coefficient of 1.089 for planimeter measurements was significant at the 5 percent significance level. This indicates a positive bias which increases as the size of field increases.

The regression coefficient of .925 for ASC measurements was significant at the 5 percent significance level and indicates a negative bias. Further, the negative bias in the ASC measurements gets larger as the size of field increases.

Although y intercept values different from zero were obtained for both the planimeter and ASC measurements, it would be logical to expect a regression through the origin. These results are consistent with the method of analysis presented previously.

The cost of obtaining measurements by the four measuring schemes differ considerably. The most accurate method, chain measurements, requires considerable work by a two-man team in each sample field. An outline of the field is accurately sketched and the area divided into regularly shaped areas (rectangles, parallelograms and triangles). These areas are then measured, using a surveyor's chain, and the dimensions recorded. From these measured areas the acreage of the field is calculated.

Planimeter and rotometer measurements can be made more easily. The field is visited and the area delineated on an aerial photograph taken of the general area. The two measurements differ only in the measurement instrument used to calculate the delineated area. The rotometer is faster to use than the planimeter.

The farmers' estimate is the simplest of all the measurements to obtain. The farmer is interviewed and asked his estimate of the acreage. An interview with the farmer is a necessary preliminary to obtaining information under any of the measuring systems considered.
Figure 1. Comparison of farmers' estimates and chain measurements.
Figure 2. Comparison of planimeter measurements and chain measurements.
Figure 3. Comparison of ASC measurements and chain measurement.
CHAPTER VI

CONCLUSIONS, RECOMMENDATIONS AND SUMMARY

From the result of farmers' estimates having almost zero bias compared with chain measurements, this relatively inexpensive method of obtaining sample field acreage would seem to be very practical for estimating cotton acreage by the USDA. However, the varying results county to county indicate that the sample was too small to state conclusively that only farmers' estimates be used to estimate acreage for an entire population. Further study should be devoted to chain measurements and farmers' estimates as a means of making acreage estimates. A double sampling scheme is recommended whereby a large sample of fields would be selected and farmers' estimates obtained followed by the selection of a subsample of these fields for chain measurement.

The large positive bias resulting from the use of planimeter measurements on aerial photographs indicate that this method is less desirable than either farmers' estimates or chain measurements. The county-to-county variability indicates that planimeter readings as a method of measuring fields is highly biased when the fields are large and/or of complex shape. This method should not be entirely discarded, however; several improvements in technique are recommended which merit further study because of the low cost of using this method relative to chain measurements. The recommendations are as follows:

1. Use of larger scale photographs so that terraces and other details can be seen more distinctly.
2. More careful delineation of the fields with narrower lines for the field boundaries than were used on photos in this study.
3. Careful attention and measurement of deductions within the fields.
4. Further experimental work with the measuring instrument, particularly in regard to measurement errors between and within operators of the instrument.
An attempt should be made to eliminate or at least reduce as much as possible the observed bias in this method of measuring to secure more conclusive evidence.

The large negative bias associated with ASC measurements in the data collected indicates that this method is also undesirable in estimating acreage. County-to-county variability in this case implies nonuniform confirmation to ASC regulations by county offices or variability in the use of measuring instruments. Further investigations should be made in methods used by different ASC county offices to determine field acreages. Particularly tests should be conducted on the rotometer (which is supposed to give the official ASC measurement) with regard to measurement errors.

Significance tests of the hypotheses that there were no differences between methods of measurement could not be made with very much confidence within counties because of the few degrees of freedom. For the combined three county total, significant values of "t" for both planimeter and ASC measurements compared with chain measurements give credence to the conclusions drawn after examining the biases.

The use of sign tests revealed some interesting results from which the following conclusions can be drawn: (1) the farmers underestimated their acreage to the extent that if 20 samples were drawn from the same population, in 19 of the samples the percentage of farmers who would overestimate their acreage would not reach 50 percent. (2) Only in one sample out of 100 samples drawn from the same population using the same procedure for marking photos and making planimeter measurements would the percentage of overestimates exceed 50 percent.

The sign test confirmed results of planimeter readings compared to chain measurements. The results noted for farmers' estimates compared to chain measurements would lead one to be careful in using the farmers' estimate as a method of estimating acreage alone. That further research in the four methods of measurement is necessary before definite conclusions can be drawn and firm recommendations made, is substantiated by the results.
From the investigation of variance components, chain measurements are the most accurate in two of the three counties and for the combined three counties. Although cost consideration was not taken into account, the other three methods of measurement cannot be discarded because of the relatively high cost of obtaining measurements by chain. The fact that between and within team variance for chain measurements was so small indicates that this phase of the work can be ignored in future studies.

**SUMMARY**

This study was carried out for the purpose of contrasting three relatively inexpensive but possible biased methods of acreage estimation with a much more expensive but assumed to be unbiased method. The stimulus for the project was the assertion that poor acreage estimates and inaccurate yield-per-acre forecasts in the summer of 1951 cost the nation's farmers an estimated $125 million. The field data were taken from a random cluster sample of 60 cotton fields drawn from Cleveland, Iredell and Union counties in North Carolina.

The four methods of field measurement considered were:

1. Chain measurements (assumed to be unbiased).
2. Farmers' estimates.
3. Planimeter measurements (from aerial photographs).
4. ASC measurements.

Analysis of the data revealed that the most accurate measurements were made by chain; the planimeter measurements overestimated acreage by .30 acres per field on the average and were significantly biased at the 5 percent significance level; ASC measurements underestimated acreage by .23 acres per field on the average and were significantly biased at the 1 percent level; farmers' estimates underestimated acreage by only .04 acres per field on the average. However, 63.3 percent of the
farmers underestimated their field acreage and a proportion test that this percentage was .50 in the population was significant at the 5 percent level.

The sample size was too small to arrive at definite conclusions on the basis of this work, but recommendations were made to continue study of all the methods on a larger sample. Specifically a double sampling procedure was proposed which would include a relatively large sample of fields on which farmers' estimates would be obtained and a subsample of those fields (approximately 100 is suggested) on which the other measuring schemes would be repeated. More refined methods were suggested for measuring fields by planimeter.
LITERATURE CITED


Airth, John Malcolm. A Comparison of Four Methods of Measuring Acreage, (under the direction of Alva Leroy Finkner).

This study was carried out to contrast three relatively inexpensive but possibly biased methods of acreage estimation with a much more expensive but assumed unbiased method.

The four methods of field measurement considered were:

1. Chain measurements (assumed unbiased)
2. Farmers' estimates
3. Planimeter measurements (from aerial photographs)

Analysis of the data revealed that the most accurate measurements were made by chain; on the average, planimeter measurements overestimated acreage by 7 percent, ASC measurements underestimated by 6 percent, farmers' estimates underestimated by 1 percent.

The sample size was too small to arrive at definite conclusions but recommendations were made

1. To continue the study of all methods, using a larger sample.
2. To investigate more refined methods of measuring fields on aerial photographs.

If the low bias of the farmers' estimates is substantiated by the larger sample, farmers' estimates and a chain measured subsample appear to be feasible sampling procedure.