Compass Plots: A Joint Graphical Representation of the Factorial Design, Treatment Means, and Fitted Effects

Kamon Budsaba  Charles E. Smith  Jim E. Riviere*

North Carolina State University, Raleigh, NC 27695

A new graphical method, called “Compass Plots”, is introduced for displaying the results in the design of experiments, especially for balanced factorial experiments. In contrast to the classical method, this plot allows statistical inferences for treatment and factor effects. Examples of $2^2$, $2^3$, and $2^4$ factorial experiments are considered here. This idea can also be extended to $2^p$ factorial designs. Several applications of compass plots, e.g. in cutaneous toxicology, are also presented.

Key Words: Bonferroni method; design of experiments; inner polygon; Kimball inequality; outer polygon; star plot.

*Kamon Budsaba is Doctoral Student, Department of Statistics, North Carolina State University, Raleigh, NC 27695-8203 (E-mail: kbudsab@stat.ncsu.edu); Charles E. Smith is Associate Professor, Department of Statistics, North Carolina State University, Raleigh, NC 27695-8203 (E-mail: bmasmith@stat.ncsu.edu); and Jim E. Riviere is Professor, Cutaneous Pharmacology and Toxicology Center, 4700 Hillsborough Street, Raleigh, NC 27606 (E-mail: Jim.Riviere@ncsu.edu)
1. INTRODUCTION

An important task of a statistician is to effectively communicate the analyzed data from a research project. Some statistical results may be presented in the flow of the text or in a table but often graphs or other figures can communicate more clearly than text can. Results from a factorial experiment are usually presented in an ANOVA table and graphs, e.g. means plots. Means plots cannot easily display the magnitude of the effects and sometimes can also be misleading when factors are qualitative data. The method of presentation that will be the most clear for a given audience needs to be determined. A new method, called “Compass Plots”, can make more dramatic comparisons in the sizes of treatment and factor effects. It emphasizes the deviation of each treatment mean and of each fitted effect from the grand mean.

The main objective of this study was to create a graphical method for presentation of the results from a $2^p$ balanced factorial experiment that can illustrate treatment and factor effects under a fixed effects model (Model I). Radial plots for multivariate data have been introduced as a descriptive tool (star and profile plots by Chambers et al. (1983), glyphs by Toit et. al. (1986), florigraph by Barton (1992), and Kiviat diagrams (Douglas, 1994)), however our compass plot also indicates the design of the experiment and allows statistical inferences. Our graphical significance test, with a fixed experimentwise error rate, for treatment effects is a similar calculation to that used in the analysis of means (ANOM) by Ott (1983) and Ramig (1983), but the compass plot can also indicate significance of factor effects.

A compass plot is a radial plot with the magnitude of each treatment mean, or of each fitted effect plus grand mean, determining the length of the ray. The rays are ordered in accordance with the experimental design. To fix the ordering in two level factorial experiments, we use the ordering in Yates algorithm. Said another way, the compass plot is essentially a star plot with the variables
being either the treatment mean or the fitted effect plus grand mean. Simultaneously confidence intervals are then added to produce inner and outer polygons.

2. METHODS

2.1 Investigate Treatment Effects - Model I

The idea of detecting treatment effects by a compass plot is to compare all treatment means with the grand mean. Following Neter et al. (1990), consider the single-factor ANOVA cell means model in a balanced experiment as follows:

\[ Y_{ij} = \mu_i + \epsilon_{ij} \]  

(1)

where:

- \( Y_{ij} \) is the value of the response variable in the jth trial for the ith treatment or factor level
- \( \mu_i \) is the parameter representing the mean of the ith treatment
- \( \epsilon_{ij} \) is the independent random error term and \( N(0, \sigma^2) \)
- \( i = 1, \ldots, k \); \( j = 1, \ldots, n \)

The factor effects model, given below, is an alternative equivalent formulation of the cell means model.

\[ Y_{ij} = \mu + \tau_i + \epsilon_{ij} \]  

(2)

where \( \tau_i = \mu_i - \mu \) is the effect of the ith treatment and \( \mu \) is the grand mean. This definition implies that \( \sum_{i=1}^{k} \tau_i = 0 \). To test for equality of treatment means, the hypotheses are as follows:
\( H_0: \tau_1 = \tau_2 = \ldots = \tau_{k-1} = 0 \)

\( H_a: \) not all \( \tau_i \) equal zero

For \( i = 1, \ldots, k-1 \), let contrast \( i \) be \( L_i = \mu_i - \mu \). An unbiased estimator of \( L_i \) is \( \hat{L}_i = \bar{Y}_i - \bar{Y} \).

Under model I; \( \text{Var}(\hat{L}_i) = (k-1/k)\text{Var}(\bar{Y}_i) = (k-1)(\sigma^2)/(k)(n) \). The Bonferroni inequality then implies that the confidence coefficient is at least \( 1-\alpha \) that the following confidence limits for the \( k-1 \) linear combinations \( L_i \) are all correct;

\[
\hat{L}_i \pm t_{1-\frac{\alpha}{2(k-1)}; k(n-1)}s(\hat{L}_i)
\]

(3)

where \( s^2(\hat{L}_i) = (k-1)(MSE)/(k)(n) \).

These confidence intervals will be smaller than those from the Scheffe' method since the number of contrasts to be estimated is less than the number of factor levels (Neter et al. 1990, p 588-589).

Under \( H_0 \), it implies that for each \( i, i = 1, \ldots, k-1 \), after some rearranging of terms:

\[
\bar{Y} - t_{1-\frac{\alpha}{2(k-1)}; k(n-1)}s(\hat{L}_i) \leq \bar{Y}_i \leq \bar{Y} + t_{1-\frac{\alpha}{2(k-1)}; k(n-1)}s(\hat{L}_i)
\]

Hence the plot of the treatment means along with its lower and upper limits as inner and outer polygons respectively, can be used in the compass plot to detect the significance of treatment effects.

### 2.2 Investigate Factor Effects - Model I

To detect the factor effects by a compass plot, each fitted effect is represented as a deviation from the grand mean. The mean response for a given treatment in a balanced two-factor study is denoted by \( \mu_{ij} \), where \( i \) refers to the level of factor A (\( i = 1, \ldots, a \)) and \( j \) refers to the level of factor B (\( j = \).
1, ..., b. Let define:

\[ \mu_i = \frac{\sum_{j=1}^{b} \mu_{ij}}{b} \]  

(4)

\[ \mu_j = \frac{\sum_{i=1}^{a} \mu_{ij}}{a} \]  

(5)

\[ \mu_{..} = \frac{\sum_i \sum_j \mu_{ij}}{ab} = \frac{\sum_i \mu_i}{a} = \frac{\sum_j \mu_j}{b} \]  

(6)

Now consider the factor effects ANOVA model for two-factor studies:

\[ Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \]  

(7)

where:

- \( Y_{ijk} \) is the value of the response variable in the kth trial when factor A is at the ith level and factor B is at the jth level.
- \( \mu_{..} \) is an overall constant representing the grand mean.
- \( \alpha_i = \mu_i - \mu_{..} \) are constants and \( \sum_i \alpha_i = 0 \) (main effect of factor A at ith level).
- \( \beta_j = \mu_j - \mu_{..} \) are constants and \( \sum_j \beta_j = 0 \) (main effect of factor B at jth level).
- \( (\alpha\beta)_{ij} = \mu_{ij} - (\mu_{..} + \alpha_i + \beta_j) = \mu_{ij} - \mu_{..} - \mu_j + \mu_{..} \) are constants and \( \sum_i (\alpha\beta)_{ij} = 0, \sum_j (\alpha\beta)_{ij} = 0 \) (interaction effect when factor A is at the ith level and factor B is at the jth level).
- \( \epsilon_{ij} \) are independent random error terms and \( N(0, \sigma^2) \).

Because of the way factorial effects sum to 0 over levels of any factor involved, estimating one effect of each type is sufficient to test the hypothesis in a two level experiment. To test for main effects and interactions in the case of a \( 2^2 \) factorial, the hypotheses are as follows:
\[ H_0 : \alpha_2 = \beta_2 = (\alpha \beta)_{22} = 0 \]

\[ H_a : \text{not all of them equal zero} \]

Consider the following contrasts; \( L_1 = \alpha_2 = \mu_2 - \mu \), \( L_2 = \beta_2 = \mu_2 - \mu \), and \( L_3 = (\alpha \beta)_{22} = \mu_{22} - (\mu + \alpha_2 + \beta_2) = \mu_{22} - \mu_2 - \mu + \mu_2 \). Least squares estimates of each \( L \) are; \( \hat{L}_1 = \bar{Y}_2 - \bar{Y} \), \( \hat{L}_2 = \bar{Y}_2 - \bar{Y} \), and \( \hat{L}_3 = \bar{Y}_{22} - \bar{Y}_2 - \bar{Y} + \bar{Y} \). Since these contrast are orthogonal, the Kimball inequality then implies that the confidence coefficient is at least \( 1 - \gamma \) that the following confidence limits for the 3 linear combinations \( L_i \), \( i = 1, 2, 3 \), are all correct:

\[
\hat{L}_i \pm t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_i) \tag{8}
\]

where \( \gamma = 1 - (1 - \gamma_i)^3 \), and \( s^2(\hat{L}_i) = MSE/2^2n \). Under \( H_0 \), after some rearranging of terms, it implies:

for the A main effect, \( \bar{Y}_2 - t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_1) \leq \bar{Y}_2 \leq \bar{Y}_2 + t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_1) \);

for the B main effect, \( \bar{Y}_2 - t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_2) \leq \bar{Y}_2 \leq \bar{Y}_2 + t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_2) \);

for the AB interaction, \( \bar{Y}_2 - t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_3) \leq \bar{Y}_2 \leq \bar{Y}_2 - t_{1 - \frac{\gamma}{2}; k(n-1)} s(\hat{L}_3) \).

Note that the quantity that is compared with the lower and upper limits above is the fitted effect \( \hat{E} \) plus or minus the grand mean, i.e.; \( \bar{Y}_2 = \hat{\alpha}_2 + \hat{\mu}_2 \), \( \bar{Y}_2 = \hat{\beta}_2 + \hat{\mu}_2 \), and \( \bar{Y}_2 + \bar{Y}_2 - \bar{Y}_{22} = -\alpha \beta_{22} + \hat{\mu}_2 \).

Since the lower and upper limits have the same amount of deviation from the grand mean, we can then use only the fitted effect plus the grand mean, which is the predicted value of \( Y \) when only that effect is considered in the model \( (\hat{Y}_E) \), in the compass plot. i.e.; \( \hat{Y}_A = \hat{\alpha}_2 + \hat{\mu}_2 \), \( \hat{Y}_B = \hat{\beta}_2 + \hat{\mu}_2 \), and \( \hat{Y}_{AB} = \hat{\alpha} \beta_{22} + \hat{\mu}_2 \). Hence plotting \( \hat{Y}_E \) along with its lower and upper limits as inner and outer polygons, can be used in the compass plot to detect the significance of factor effects. Our procedure is reminiscent of the use of Yates algorithm to generate, for the high levels of all factors, one fitted
effect of each type for a $2^p$ dataset. (Vardeman 1994, p 483)

In summary, To detect the treatment effects, we plot each treatment mean as a ray on the compass plot. Vertex order follows the Yates algorithm. Then we construct contrasts to compare each treatment mean with the grand mean. After setting an appropriate experimentwise error rate, we then construct a simultaneous confidence intervals by the Bonferroni method to get the inner and outer bound around the grand mean. If all treatment means are inside this band, it indicates no significant difference among treatments. Similarly, to detect the factor effects, the grand mean plus each fitted effect is then used instead of the treatment means, and the Kimball Inequality is used to construct the simultaneous intervals. If any effect is outside the band, it implies that effect is significant.

Above, we used the inner and outer polygons in the factor effects compass plot to detect significant interactions. We can also qualitatively determine the presence or absence of interactions by using the compass plot for treatment means and “squareness” in the way parallelism is used in the usual means plot. Figures 1 and 2 are the means plot and compass plot respectively when no interaction is present. The parallelism in the means plot becomes a square in compass plot by using a folding technique. Lack of parallelism or presence of interaction as shown in Figures 3 and 5 becomes an inside rectangle for non-crossover interaction and an outside rectangle for crossover interaction (Gail et al. 1985) as shown in Figures 4 and 6 respectively.

3. EXAMPLES

Example 1: $2^2$ Factorial Experiment (Steel et al. 1997)

Factor A: Time of bleeding (morning or afternoon)

Factor B: Diethylstilbestrol - an estrogenic compound (with or without)
Response: Plasma phospholipid in lambs

Five lambs were assigned at random to each of four treatment groups. Treatment means and ANOVA table are shown below:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(1)</th>
<th>a</th>
<th>b</th>
<th>ab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>13.28</td>
<td>36.53</td>
<td>19.36</td>
<td>27.81</td>
</tr>
</tbody>
</table>

(Grand Mean = 24.246)

ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>3</td>
<td>1,539.4066</td>
<td>513.1355</td>
<td>21.61</td>
<td>.0001*</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>1,256.7466</td>
<td>1,256.7466</td>
<td>52.93</td>
<td>.0001*</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>8.7120</td>
<td>8.7120</td>
<td>0.37</td>
<td>.5532</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>273.9480</td>
<td>273.9480</td>
<td>11.54</td>
<td>.0037*</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>379.9233</td>
<td>23.7452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>1,919.3299</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For treatment effects; $s^2(\hat{L}_i) = (k - 1)(MSE)/(k)(n) = (3)(23.7452)/(4)(5) = 3.5618$. If $\alpha = .05$, by the Bonferroni method, $t_{1-\frac{.05}{(2)(3)}:16} = t_{.9917:16} = 2.6750$. Hence, for $i = 1,2,3$:

$$24.246-(2.6750)\sqrt{3.5618} \leq \bar{Y}_i \leq 24.246+(2.6750)\sqrt{3.5618}$$

$$19.1976 \leq \bar{Y}_i \leq 29.2944.$$

The compass plot for the treatment effects is shown in Fig. 7. Only treatments (1) and a, are significantly different from the grand mean.
For factor effects, the fitted effects, \( \hat{E} \), and \( \hat{YE} \) can be obtained as follows:

\[
\begin{array}{c|ccc}
\text{Mean} & A & B & AB \\
\hline
(1) = 13.278 & - & - & + \\
a = 36.534 & + & - & - \\
b = 19.360 & - & + & - \\
ab = 27.812 & + & + & + \\
\hat{E} & 7.927 & -0.660 & -3.701 \\
\hat{YE} & 32.173 & 23.586 & 20.545 \\
\end{array}
\]

For \( i = 1,2,3, \) \( s^2(\hat{L}_i) = MSE/2^2n = 23.7452/(4)(5) = 1.1873. \) If \( \gamma = .05. \) By the Kimball inequality, \( \gamma_i = .01695 \) and hence \( t_{1-.01695/16} = t_{.9915/16} = 2.6647. \) Then for each factor effect:

\[
24.246-(2.6647)\sqrt{1.1873} \leq \hat{Y}_E \leq 24.246+(2.6647)\sqrt{1.1873}
\]

\[
21.3425 \leq \hat{Y}_E \leq 27.1495
\]

The compass plot for the factor effects is shown in Fig. 8. Factor A and interaction AB are significant effects.

**Example 2**: \( 2^2 \) Factorial Experiment (Neter et al. 1990)

Factor A : Gender (male or female)

Factor B : Body fat (low or high)

Factor C : Smoking history (light or heavy)

Response : The exercise tolerance stress test which is measured in minutes until fatigue occurs while the subject is performing on a bicycle apparatus.

Three subjects in each treatment combination were studied. Treatment means and ANOVA table are shown below:
Treatment | (1) a b ab c ac bc abc
Mean  | 26 20 14 12 20 12 16 10

(Grand Mean = 16.25)

ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>7</td>
<td>610.50</td>
<td>87.2143</td>
<td>9.97</td>
<td>.0001*</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>181.50</td>
<td>181.50</td>
<td>20.74</td>
<td>.0003*</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>253.50</td>
<td>235.50</td>
<td>28.97</td>
<td>.0001*</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>13.50</td>
<td>13.50</td>
<td>1.54</td>
<td>.2321</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>73.50</td>
<td>73.50</td>
<td>8.40</td>
<td>.0105?</td>
</tr>
<tr>
<td>AC</td>
<td>1</td>
<td>13.50</td>
<td>13.50</td>
<td>1.54</td>
<td>.2321</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>73.50</td>
<td>73.50</td>
<td>8.40</td>
<td>.0105?</td>
</tr>
<tr>
<td>ABC</td>
<td>1</td>
<td>1.50</td>
<td>1.50</td>
<td>0.17</td>
<td>.6843</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>140.00</td>
<td>8.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>750.50</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For treatment effects; $s^2(\hat{L}_i) = (k - 1)(MSE)/(k)(n) = (7)(8.75)/(8)(3) = 2.5521$. If $\alpha = .05$, by the Bonferroni method, $t_{1 - \frac{.05}{(2)(7)};16} = t_{.9964;16} = 3.0821$. Hence, for $i = 1, \ldots, 7$:

$$16.25 - (3.0821)\sqrt{2.5521} \leq \bar{Y}_i \leq 16.25 + (3.0821)\sqrt{2.5521}$$

$$11.3263 \leq \bar{Y}_i \leq 21.1737.$$  

The compass plot for treatment effects is shown in Fig. 9. Only treatments (1) and abc, are significantly different from the grand mean.
For factor effects, the fitted effects, \((\hat{E})\), and \(\hat{Y}_E\) can be obtained as follows:

<table>
<thead>
<tr>
<th>Mean</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>C</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)=26</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>a =20</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>b =14</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ab=12</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c =20</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ac =12</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>bc =16</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>abc=10</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

\[
\hat{E} = -2.75, -3.25, 0.75, -1.75, -0.75, 1.75, -0.25 \\
\hat{Y}_E = 13.50, 13.00, 17.00, 14.50, 15.50, 18.00, 16.00
\]

For \(i = 1, \ldots, 7\); 
\[s^2(\hat{L}_i) = MSE/2^3n = 8.75/(8)(3) = 0.3646.\] If \(\gamma = .05\), by the Kimball inequality, \(\gamma_i = .0073\) and hence \(t_{1-.0073;16} = t_{.9964;16} = 3.0716\). Then, for each effect:

\[
16.25-(3.0716)\sqrt{0.3646} \leq \hat{Y}_E \leq 16.25+(3.0716)\sqrt{0.3646} \\
14.39355 \leq \hat{Y}_E \leq 18.1047
\]

The compass plot for the factor effects is shown in Fig. 10. Factors A and B are significant effects.

**Example 3**: \(2^{4-1}\) Fractional Factorial Experiment (Vardeman 1994)

Factor A: Side of cloth (nozzle side or opposite side)

Factor B: Yarn type (air spun or ring spun)

Factor C: Pick density (35 ppi or 50 ppi)
Factor D: Air Pressure (30 psi or 45 psi)

Response: Woven fabric tenacity (strength per unit linear density)

A replicated $2^{4-1}$ experiment was done using the generator D=ABC and five pieces of cloth were tested for each of the eight different treatment combination. Treatment means and ANOVA table are shown below:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(1)</th>
<th>ad</th>
<th>bd</th>
<th>ab</th>
<th>cd</th>
<th>ac</th>
<th>bc</th>
<th>abcd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>24.50</td>
<td>22.05</td>
<td>24.52</td>
<td>25.00</td>
<td>25.68</td>
<td>24.51</td>
<td>24.68</td>
<td>24.23</td>
</tr>
</tbody>
</table>

(Grand Mean = 24.3962)

Note that for the generator D=ABC, the grand mean ($\bar{Y}_{..}$) estimates $\mu_{..} + \alpha \beta \gamma \delta_{2222}$, rather than $\mu_{..}$.

ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>7</td>
<td>38.322</td>
<td>5.4747</td>
<td>4.0686</td>
<td>.0027*</td>
</tr>
<tr>
<td>A=BCD</td>
<td>1</td>
<td>8.0551</td>
<td>8.0551</td>
<td>5.9862</td>
<td>.0201?</td>
</tr>
<tr>
<td>B=ACD</td>
<td>1</td>
<td>1.7851</td>
<td>1.7851</td>
<td>1.3266</td>
<td>.2579</td>
</tr>
<tr>
<td>AB=CD</td>
<td>1</td>
<td>8.3266</td>
<td>8.3266</td>
<td>6.1880</td>
<td>.0183?</td>
</tr>
<tr>
<td>C=ABD</td>
<td>1</td>
<td>5.7381</td>
<td>5.7381</td>
<td>4.2643</td>
<td>.0471</td>
</tr>
<tr>
<td>AC=BD</td>
<td>1</td>
<td>.0766</td>
<td>.0766</td>
<td>.0569</td>
<td>.8130</td>
</tr>
<tr>
<td>BC=AD</td>
<td>1</td>
<td>11.2891</td>
<td>11.2891</td>
<td>8.3896</td>
<td>.0068*</td>
</tr>
<tr>
<td>ABC=D</td>
<td>1</td>
<td>3.0526</td>
<td>3.0526</td>
<td>2.2686</td>
<td>.1418</td>
</tr>
<tr>
<td>Error</td>
<td>32</td>
<td>43.0592</td>
<td>1.3456</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>39</td>
<td>81.3821</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For treatment effects; 

\[ s^2(\hat{L}_i) = (k - 1)(MSE)/(k(n)) = (7)(1.3456)/(8)(5) = 0.2355. \]

If \( \alpha = 0.05 \), by the Bonferroni method, 

\[
t_{1-0.05/7.32}^{0.0073} = t_{0.9964;32} = 2.8741.
\]

Hence, for \( i = 1, \ldots, 7 \):

\[
24.3962 - (2.8741)0.2355 \leq \tilde{Y}_i \leq 24.3962 + (2.8741)0.2355
\]

\[
23.0015 \leq \tilde{Y}_i \leq 25.7910
\]

The compass plot for the treatment effects is shown in Fig. 11. Only treatment ad is significantly different from the grand mean.

For factor effects, the fitted effects, \((\hat{E})\), and \(\hat{Y}_E \) can be obtained as follows:

<table>
<thead>
<tr>
<th>Mean</th>
<th>A=BCD</th>
<th>B=ACD</th>
<th>AB=CD</th>
<th>C=ABD</th>
<th>AC=BD</th>
<th>BC=AD</th>
<th>ABC=D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)=24.50</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>ad =22.05</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>bd =24.52</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ab=25.00</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>cd=25.68</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ac =24.51</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>bc =24.68</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>abcd=24.23</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(\hat{E} )</td>
<td>-0.4488</td>
<td>0.2112</td>
<td>0.4562</td>
<td>0.3788</td>
<td>0.0438</td>
<td>-0.5312</td>
<td>-0.2762</td>
</tr>
</tbody>
</table>

For \( i = 1, \ldots, 7 \); 

\[ s^2(\hat{L}_i) = MSE/2^3n = 1.3456/(8)(5) = 0.0336. \]

If \( \gamma = 0.05 \), by the Kimball inequality, \( \gamma_i = 0.0073 \) and hence 

\[
t_{1-0.0073/8;32}^{0.0073} = t_{0.9964;32} = 2.8654.
\]

Then, for each effect:

\[
24.3962 - (2.8654)0.0336 \leq \hat{Y}_E \leq 24.3962 + (2.8654)0.0336
\]
The compass plot for the factor effects is shown in Fig. 12. Factor BC=AD is significant effect.

**Example 4 :** $2^4$ Factorial Experiment (Qiao et al. 1996)

Factor A : Vehicle (acetone or dimethylsulfoxide)

Factor B : The surfactant sodium lauryl sulfate (absence or presence)

Factor C : The rubefacient methyl nicotinate (absence or presence)

Factor D : The reducing agent stannous chloride (absence or presence)

Response : The penetration of parathion over the 8-hr experimental period after dosing on isolated perfused porcine skin

Triplicate observations in each group was used to access treatment and factor effects and interactions. Treatment means and ANOVA table are shown below:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(1) a b ab c ac bc abc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.3470 3.0247 5.9783 4.4713 3.9963 1.2697 1.6337 3.9947</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment</th>
<th>d ad bd abd cd acd bcd abcd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.4797 1.2850 2.7863 4.2980 2.7917 1.4763 3.8937 2.6117</td>
</tr>
</tbody>
</table>

(Grand Mean = 3.2711)
ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>15</td>
<td>91.2819</td>
<td>6.0855</td>
<td>3.66</td>
<td>.0010*</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>10.4776</td>
<td>10.4776</td>
<td>6.30</td>
<td>.0173?</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>9.1805</td>
<td>9.1805</td>
<td>5.52</td>
<td>.0252?</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>17.4339</td>
<td>17.4339</td>
<td>10.48</td>
<td>.0028*</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>15.1965</td>
<td>15.1965</td>
<td>9.13</td>
<td>.0049*</td>
</tr>
<tr>
<td>AC</td>
<td>1</td>
<td>0.4501</td>
<td>0.4501</td>
<td>0.27</td>
<td>.6065</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>0.6062</td>
<td>0.6062</td>
<td>0.36</td>
<td>.5503</td>
</tr>
<tr>
<td>ABC</td>
<td>1</td>
<td>0.0674</td>
<td>0.0674</td>
<td>0.04</td>
<td>.8418</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>9.4341</td>
<td>9.4341</td>
<td>5.67</td>
<td>.0234?</td>
</tr>
<tr>
<td>AD</td>
<td>1</td>
<td>0.1569</td>
<td>0.1569</td>
<td>0.09</td>
<td>.7608</td>
</tr>
<tr>
<td>BD</td>
<td>1</td>
<td>0.8400</td>
<td>0.8400</td>
<td>0.50</td>
<td>.4825</td>
</tr>
<tr>
<td>ABD</td>
<td>1</td>
<td>0.8775</td>
<td>0.8775</td>
<td>0.53</td>
<td>.4729</td>
</tr>
<tr>
<td>CD</td>
<td>1</td>
<td>8.8014</td>
<td>8.8014</td>
<td>5.29</td>
<td>.0281?</td>
</tr>
<tr>
<td>ACD</td>
<td>1</td>
<td>5.4230</td>
<td>5.4230</td>
<td>3.26</td>
<td>.0804</td>
</tr>
<tr>
<td>BCD</td>
<td>1</td>
<td>0.5002</td>
<td>0.5002</td>
<td>0.30</td>
<td>.5873</td>
</tr>
<tr>
<td>ABCD</td>
<td>1</td>
<td>11.8366</td>
<td>11.8366</td>
<td>7.12</td>
<td>.0119?</td>
</tr>
<tr>
<td>Error</td>
<td>32</td>
<td>53.2343</td>
<td>1.6636</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47</td>
<td>144.5162</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For treatment effects; \( s^2(\hat{L}_i) = (k - 1)(MSE)/(k)(n) = (15)(1.6636)/(16)(3) = 0.5199. \) If \( \alpha = .10, \) by the Bonferroni method, \( t_{1-(\frac{10}{15})}^{16} = t_{0.9967;32} = 2.9016. \) Hence, for \( i = 1, \ldots, 15: \)

\[
3.2711-(2.9016)\sqrt{0.5199} \leq \bar{Y}_i \leq 3.2711+(2.9016)\sqrt{0.5199}
\]
The compass plot for the treatment effects is shown in Fig. 13. Only treatment b is significantly different from the grand mean.

For factor effects, $\hat{Y}_E$ can be obtained as in previous examples:

<table>
<thead>
<tr>
<th>$\hat{Y}_E$</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>C</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.8039</td>
<td>3.7084</td>
<td>3.8738</td>
<td>2.7084</td>
<td>3.3679</td>
<td>3.1587</td>
<td>3.3086</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\hat{Y}_E$</th>
<th>D</th>
<th>AD</th>
<th>BD</th>
<th>ABD</th>
<th>CD</th>
<th>ACD</th>
<th>BCD</th>
<th>ABCD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.8278</td>
<td>3.3283</td>
<td>3.4034</td>
<td>3.1359</td>
<td>3.6993</td>
<td>2.9350</td>
<td>3.3732</td>
<td>2.7745</td>
</tr>
</tbody>
</table>

For $i = 1, \ldots, 15$; $s^2(\hat{L}_i) = \frac{MSE}{24^n} = \frac{1.6636}{(16)(3)} = 0.0346$. If we are interested in only the main effects and twoway interactions, we have only 10 orthogonal contrasts. If $\gamma = .10$, by the Kimball inequality, $\gamma_1 = .0105$ and hence $t_{1-.0105,32} = t_{.9948,32} = 2.7225$ Then:

For each factor effects of interest:

$3.2711 - (2.7225)\sqrt{0.0346} \leq \hat{Y}_E \leq 3.2711 + (2.7225)\sqrt{0.0346}$

$2.7649 \leq \hat{Y}_E \leq 3.7773$

The compass plot for the factor effects is shown in Fig. 14. Factor C and interaction AB are significant effects.
4. CONCLUSION AND DISCUSSION

Compass plots are a useful graphical method to display the significant results not only for $2^p$ factorial experiments but also for any type of factorial or any design of experiment. For factors with more than two levels, main effects at each level, including their interactions, each with 1 df have to be considered. For example, $3^2$ factorial experiment yields a nonagon shape for treatment means and an octagon shape for the factor effects. For data where the responses are negative, adding a suitable positive constant to each response allows the compass plot to be used.

For a univariate response two-level experiment, other alternatives to the Yates vertex ordering can be used. Vertices can be ordered by the magnitude of treatment means or effects. This will give a spiral shape, c.f. the seasonal cobweb plot for time series data (Toit et al., 1986), and the significant groups will be in the same neighbor. For multi-response experiments, a fixed order, e.g. by the Yates algorithm, is better for comparison among each measurement.

We have presented results for the fixed effects model, but this method can also be extended to the mixed effects model. In the mixed model, each fixed effect has a different standard error, so we have a set of inner and outer polygons for each fixed effect. For the random effects model, interest centers on the variability of the $\mu_i$, measured by $\sigma^2_{\mu}$, and the mean of the $\mu_i$'s, $\mu$. Hence there is little interest in comparing means for the particular $\mu_i$'s in the experiment and compass plots are not very useful in this case.

If the factorial experiment is not balanced, the inner and outer polygons will be irregular, i.e. the treatments with fewer replications will have a larger confidence interval at that vertex. Heterogeneity of variances can be handled by transformations. Other choices for multiple comparisons could also be used (Neter et al., 1990), e.g. the Bonferroni method could be used instead of the Kimball inequality for factor effects simultaneous confidence intervals. In our plot, for significance testing
we used polygonal, rather than circular, curves because polygon plots are more widely available in common software, e.g., Microsoft Excel.

In summary, compass plots provide a graphical counterpart to ANOVA tables and mean plots.

ACKNOWLEDGEMENT

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REFERENCES


Figure 1. Means plot with no interaction

Figure 2. Compass plot with no interaction

Figure 3. Means plot with non-crossover interaction

Figure 4. Compass plot with non-crossover interaction

Figure 5. Means plot with crossover interaction

Figure 6. Compass plot with crossover interaction

Figure 7. Compass plot for detecting treatment effects in Example 1.

Figure 8. Compass plot for detecting factor effects in Example 1.

Figure 9. Compass plot for detecting treatment effects in Example 2.

Figure 10. Compass plot for detecting factor effects in Example 2.

Figure 11. Compass plot for detecting treatment effects in Example 3.

Figure 12. Compass plot for detecting factor effects in Example 3.

Figure 13. Compass plot for detecting treatment effects in Example 4.

Figure 14. Compass plot for detecting factor effects in Example 4.
Compass plot (No Interaction)

- Projections
- Trt. Means
Means Plot (Non-crossover Interaction)
Compass Plot (Non-Crossover Interaction)
Compass Plot (Crossover Interaction)

- Projections
- Trt. Means
Treatment Effects (Example 2.)
Factor Effects (Example 2.)
Treatment Effects (Example 3.)
Factor Effects (Example 3.)

- BC = AD
- CA = BD
- AB = CD
- B = ACD
- A = BCD
- ABC = D
- GM

Effects: •
Inner: -
GM: ----
Outer: ---
Treatment Effects (Example 4.)

- Trt. Means
- Inner
- GM
- Outer