Discussions for **HOMEWORK 3**

**EXERCISE 3.6**

This problem really wanted us to go back to the age of dinosaurs when all the fancy software like SAS, R, MINITAB etc did not exist; we only had our formulas and our calculators.

(a) So we consider the following table:

<table>
<thead>
<tr>
<th>y</th>
<th>$y^2$</th>
<th>x</th>
<th>$x^2$</th>
<th>xy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>4</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>5</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>6</td>
<td>36</td>
<td>30</td>
</tr>
<tr>
<td>18</td>
<td>68</td>
<td>21</td>
<td>91</td>
<td>78</td>
</tr>
</tbody>
</table>

Note that $\bar{x} = 3.5, \bar{y} = 3$.

So, $SS_{xy} = \sum_i x_i y_i - n \bar{x} \bar{y} = 78 - 6 \times 3.5 \times 3 = 15$

& $SS_{xx} = \sum_i x_i^2 - n \bar{x}^2 = 91 - 6 \times (3.5)^2 = 17.5$

Thus we have

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{15}{17.5} = 0.8571429$$

Also

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 3 - 0.8571429 \times 3.5 \approx 0$$
EXERCISE 3.7

Computations same as Problem 3.6; though much more easier as \( \sum x_i = 0 \) which simplifies \( SS_{xy} \) and \( SS_{xx} \) as follows

\[
SS_{xy} = \sum x_i y_i , \quad SS_{xx} = \sum x_i^2
\]

For this problem, the results are

\[
\hat{\beta}_1 = -1.2 , \quad \hat{\beta}_0 = 2
\]
EXERCISE 3.8

( a ) This part only asks you to propose a model to relate the appraised property value to the sale price. Only stating that a linear regression model of the form

\[ Y = \beta_0 + \beta_1 x + \text{error} \]

where \( x \): appraised property value, \( Y \): sale price would suffice.

However many of you have gone on to obtain a regression equation on the few values given and proposed that as a model; many have tried to guess it from the given plot. These are acceptable as well. ( NOTE: It is not advisable to put down the least squares regression equation in this part!!! )

( b ) From the scatterplot, it does seem that a linear model would be an appropriate fit to the data.

( c ) The best-fitting line ( as given in the data ) is given as

\[ \text{Sale Price} = 1.36 + 1.40827 * \text{Market Val} \]

( NOTE: Do not approximate the estimate for the slope, even though the MINITAB output states it as 1.41 in the first line. Use the value as mentioned later. This is because this rounding off may cause some error later )

( d ) Interpretation of Y-intercept: The sale price of a house whose market value is $0, is $1360. This is quite absurd as this cannot happen in real-life situations.

( e ) Interpretation of Slope: The mean sale price of a house in the said neighbourhood increases by $1.40827 ( in Thousands ) per unit increase in Market Value. That is, if the market value of houses in the neighbourhood goes up by $1000, the average sale price goes up by $1408.27 units.

( f ) Plug in \( x = 300 \) in the least squares regression. The required estimate of the mean sale price comes out to be $423841.

( NOTE: Had you used the regression equation using the approximated values of the coefficients, this result would come out to be $424400, which is $559 more than what becomes the actual estimate. Is this deviation large??? Calculate the relative error and decide for urself! ☺️ )
EXERCISE 3.12

(a) \( y : number\ of\ strikes, \quad x : age\ of\ fish \)

A straight line model relating \( y \) and \( x \) can be written as

\[
y = \beta_0 + \beta_1 x + \text{error}
\]

(b) The least squares prediction equation is

\[
\hat{y} = 175.7033 - 0.8195 x
\]

(c) \( \hat{\beta}_0 = 175.7033 \) denotes the average number of strikes of a fish aged 0 years. This is not feasible as there such a concept cannot be associated with a fish who is not yet born.

(d) \( \hat{\beta}_1 = -0.8195 \) is the mean increase in the number of strikes of a fish when its age is increased by 1 year. In other words, the mean number of strikes of the fishes decreases on an average by 0.8195 per unit increase in the age of the fishes.