Double-Sampling Acceptance Plans

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1 Setup

A double-sampling acceptance plan is characterized by two sample sizes $n_1$ and $n_2$, two acceptance numbers $c_1$ and $c_2$, and two rejection numbers $r_1$ and $r_2$.

1.1 Decisions

A sample of $n_1$ items is inspected, and based on the number $d_1$ of nonconforming items, one of three decisions is made:

- If $0 \leq d_1 \leq c_1$, the lot is accepted;
- If $d_1 \geq r_1$, the lot is rejected;
- If $c_1 < d_1 < r_1$, a second sample of size $n_2$ items is inspected.

If the second sample is taken, based on $d_1$ and the number $d_2$ of nonconforming items in the second sample:

- If $d_1 + d_2 \leq c_2$, the lot is accepted;
- If $d_1 + d_2 \geq r_2$, the lot is rejected.

To ensure that the lot is either accepted or rejected, $r_2 = c_2 + 1$. Often, $r_1$ is also taken to be equal to $r_2 = c_2 + 1$, but that is not required. The calculations below are for the more general case.
2 Acceptance Probability

The acceptance probability is

\[ P(\text{accept}) = \sum_{d_1=0}^{n_1} P(\text{accept and } D_1 = d_1) \]

\[ = \sum_{d_1=0}^{n_1} P(\text{accept } | \ D_1 = d_1)P(D_1 = d_1). \]

2.1 First sample

For \(0 \leq d_1 \leq c_1\),

\[ P(\text{accept } | \ D_1 = d_1) = 1 \]

and for \(r_1 \leq d_1 \leq n_1\),

\[ P(\text{accept } | \ D_1 = d_1) = 0 \]

so

\[ P(\text{accept}) = \sum_{d_1=0}^{c_1} P(D_1 = d_1) + \sum_{d_1=c_1+1}^{r_1-1} P(\text{accept } | \ D_1 = d_1)P(D_1 = d_1) \]

\[ = P(\text{accept on 1st sample}) + P(\text{accept on 2nd sample}) \quad (1) \]

where

\[ P(\text{accept on 1st sample}) = \sum_{d_1=0}^{c_1} P(D_1 = d_1) \quad (2) \]

and

\[ P(\text{accept on 2nd sample}) = \sum_{d_1=c_1+1}^{r_1-1} P(\text{accept } | \ D_1 = d_1)P(D_1 = d_1). \quad (3) \]
2.2 Second sample

For $c_1 + 1 \leq d_1 \leq r_1 - 1$,

$$P(\text{accept} \mid D_1 = d_1) = \sum_{d_2=0}^{r_2} P(\text{accept} \mid D_1 = d_1 \text{ and } D_2 = d_2) P(D_2 = d_2).$$

Now

$$P(\text{accept} \mid D_1 = d_1 \text{ and } D_2 = d_2) = \begin{cases} 1 & 0 \leq d_1 + d_2 \leq c_2 \\ 0 & r_2 \leq d_1 + d_2 \end{cases}$$

so

$$P(\text{accept} \mid D_1 = d_1) = \sum_{d_2=0}^{c_2-d_1} P(D_2 = d_2) \quad (4)$$

2.3 Overall

Combine equations (1), (2), (3), and (4) to get

$$P(\text{accept}) = \sum_{d_1=0}^{c_1} P(D_1 = d_1) + \sum_{d_1=c_1+1}^{r_1-1} P(D_1 = d_1) \sum_{d_2=0}^{c_2-d_1} P(D_2 = d_2).$$

All of the probabilities on the right hand side are binomial probabilities:

$$P(D_i = d_i) = \binom{n_i}{d_i} p^{d_i} (1-p)^{n_i-d_i},$$

so the computation is straightforward, if tedious except when the acceptance and rejection numbers are small.

2.3.1 Special Case

Note that if $r_1 = c_1 + 2$, the second sample is taken only if $d_1 = c_1 + 1 = r_1 - 1$, and the sum in

$$P(\text{accept on 2nd sample})$$

has only a single term, so

$$P(\text{accept}) = \sum_{d_1=0}^{c_1} P(D_1 = d_1) + P(D_1 = c_1 + 1) \sum_{d_2=0}^{c_2-c_1-1} P(D_2 = d_2).$$
If in addition the rejection numbers $r_1$ and $r_2$ are equal, say $r_1 = r_2 = r$, then

\[ c_2 - c_1 - 1 = r - 1 - (r - 2) - 1 = 0, \]

so

\[ P(\text{accept}) = \sum_{d_1=0}^{c_1} P(D_1 = d_1) + P(D_1 = c_1 + 1)P(D_2 = 0). \]