What is Experimental Design?

A designed experiment is a series of tests in which certain variables (or *factors*) are changed systematically, while others are held constant, while observing a response variable $Y$.

Some factors are *controllable*, both in test conditions and in routine operation.

Others are *uncontrollable* in routine operation, though they may be controllable in test conditions. These are also called *noise factors*. 
Objectives

- Find most influential factors.
- Find settings of influential factors to achieve a target value for $E(Y)$.
- Find settings of influential factors to reduce variability of $Y$.
- Find settings of influential factors to reduce impact of uncontrollable factors on $Y$. 

What is Experimental Design?
Examples of Designed Experiments

Characterizing a process
Soldering components to a printed circuit board

Controllable factors:
- Solder temperature;
- Preheat temperature;
- Conveyor speed;
- Flux type;
- Flux specific gravity;
- Solder wave depth;
- Conveyor angle.
Uncontrollable factors (controllable in a test):

- PC board thickness;
- Component types;
- Component layout;
- Operator;
- Production rate.

The relevant quality characteristic is the number of defective joints on a board. **Characterizing** the process means identifying the influential factors, and describing their impact on the response.

With this many factors, often only very few are found to be influential. A **screening experiment** is often used to find them.
Optimizing a process

Improving the yield of a chemical process.

Time and temperature are found to be influential. An experiment with a factorial design can give a detailed picture of their joint impact.
Example

A chemical process runs at around 75% yield. A $2 \times 2$ factorial experiment suggests that increasing the temperature and decreasing the time would improve the yield:
Guidelines for Designing Experiments

Steps in designing an experiment

1. Recognize and state the problem;
2. Choose factors and their levels in the experiment; often initially many factors are identified, then screened;
3. Select the response variable, including how it is to be measured; consider gauge capability;
4. Choose the experimental design; factorial design, full, replicated, or fractional replicated; not one-factor-at-a-time;
5. Perform the experiment; adhere to protocol, randomization;
6. Analyze the data;
7. Summarize the conclusions and recommendations.
Factorial Experiments

In a factorial design, each experimental factor has a small number of levels; that is, the actual settings that will be used.

For a given run, use a specific combination of levels, defining the treatment.

If factor $A$ has $a$ levels, factor $B$ has $b$ levels, ..., then there are $a \times b \times \ldots$ treatments.

For example, if 12 factors each have 2 levels, there are $2^{12} = 4096$ treatments.
Example: Priming aircraft parts
Response: adhesion of finish coat to primer

Factors:
- Method (A): dipping or spraying ($a = 2$ levels);
- Primer (B): three different primers ($b = 3$ levels).

The complete design (all treatments) was replicated $n = 3$ times, with all 18 runs carried out in random order (a completely randomized design).
Statistical model

\[ Y_{i,j,k} = \mu + \tau_i + \beta_j + (\tau \beta)_{i,j} + \epsilon_{i,j,k} \]

- \( \mu \) = overall mean
- \( \tau_i \) = effect of \( i^{th} \) level of factor \( A \)
- \( \beta_j \) = effect of \( j^{th} \) level of factor \( B \)
- \( (\tau \beta)_{i,j} \) = effect of interaction of factors \( A \) and \( B \)
- \( \epsilon_{i,j,k} \) = random error component with zero mean
In R:

```r
paint <- read.csv("Data/Table-13-01.csv")
summary(aov(Adhesion ~ factor(Primer) * Method, paint))
```

Each line in the table relates to testing the null hypothesis that the corresponding component in the statistical model is all zeros.

If the interaction terms \((\tau \beta)_{i,j}\) are all zero, the model is additive:

\[
E(Y_{i,j,k}) = \mu + \tau_i + \beta_j.
\]

Note that the interaction term is not significant, so the factors appear to have additive effects.
The **interaction plot** gives a graphical view of interactions.

```r
with(paint, interaction.plot(Primer, Method, Adhesion))
```

When effects are additive, the traces are parallel.

When effects are *not* additive, the traces show the nature of the interaction: how the effect of one factor changes with the level of the other.
Fitted values and residuals

The parameters in the statistical model are estimated by least squares, with some constrained to be zero:

```
summary(lm(Adhesion ~ factor(Primer) * Method, paint))
```

The fitted values and the residuals are

\[
\hat{y}_{i,j,k} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + (\hat{\tau} \hat{\beta})_{i,j}
\]
\[
e_{i,j,k} = y_{i,j,k} - \hat{y}_{i,j,k}.
\]

Because the data are \textit{balanced},

\[
\hat{y}_{i,j,k} = \bar{y}_{i,j}.
\]
Make the usual plots of the residuals:

\[
\text{plot(lm(Adhesion} \sim \text{factor(Primer) } * \text{Method, paint))}
\]

The first plot, residuals versus fitted values, is not very informative. Each column of plotted points is necessarily centered at zero, so the means cannot show any pattern. If the \textit{dispersion} changes with the fitted value, the points will show a wedge shape, but the third plot shows such effects better.

The second plot is the q-q plot of the residuals against the normal distribution, and can be \textit{very} informative.
The third plot shows the *absolute values* of the residuals (actually, their square roots) versus the fitted values, and this shows more clearly any tendency for the dispersion of the residuals to change with the fitted value.

The fourth plot shows different aspects of the residuals for different designs. In this case, it shows the residuals grouped by factor levels, which can reveal a combination of factor levels that affects the dispersion.