Double, Multiple, and Sequential Sampling

Double-sampling

In a double-sampling plan, a first sample of size $n_1$ is inspected, revealing $d_1$ nonconforming items, and one of three decisions is made:

- If $d_1$ is small ($d_1 \leq c_1$ for a first sample acceptance number $c_1$), accept the lot;
- If $d_1$ is large ($d_1 > c_2$ for a combined sample acceptance number $c_2$), reject the lot;
- Otherwise ($c_1 < d_1 \leq c_2$), inspect a second sample of size $n_2$.

If the second sample has $d_2$ nonconforming items, accept the lot if $d_1 + d_2 \leq c_2$ and reject it if $d_1 + d_2 > c_2$. 
Rejection numbers

In general, we could specify *rejection numbers* $r_1$ and $r_2$, with the rules:

- If $d_1 \geq r_1$, reject the lot based on the first sample;
- If $d_1 + d_2 \geq r_2$, reject the lot based on both samples.

Implicitly, above we used $r_1 = r_2 = c_2 + 1$. Of course, if we specify $r_2$, it *must* be $c_2 + 1$, to ensure we reach a decision.
In R:

OC curve for the double-sampling plan with 
\( n_1 = 50, c_1 = 1, n_2 = 100, c_2 = 3 \):

```
library(AcceptanceSampling)
pd <- seq(from = 0, to = 0.12, length = 50)
plot(OC2c(n = c(50, 100), c = c(1, 3), r = c(4, 4), pd = pd),
     type = "l")
```
For a double-sampling plan, \( OC2c() \) requires you to specify rejection numbers in the argument \( r \).

In a single-sampling plan, \( r \) defaults to \( c + 1 \), and *must* be \( c + 1 \) if it is given explicitly.

To match Montgomery’s description, the rejection number at each stage should be \( c_2 + 1 \), which should be the default in \( OC2c() \), but currently is not.
OC curves again

Montgomery suggests showing the probabilities of acceptance and rejection at the first sample, in addition to the overall probability of acceptance.

In R:

```r
par(mar = .1 + c(5, 4, 4, 4))
plot(OC2c(n = c(50, 100), c = c(1, 3), r = c(4, 4), pd = pd),
     type = "l")
lines(pd, OC2c(n = 50, c = 1, pd = pd)@paccept, lty = 2)
lines(pd, OC2c(n = 50, c = 3, pd = pd)@paccept, lty = 3)
axis(4, pretty(0:1), format(1 - pretty(0:1)))
mtext("P(reject)", 4, 2.5)
legend("topright",
       c("P(accept)",
          "P(accept on first sample)",
          "P(reject on first sample)",
          lty = 1:3)
```
Average sample number

The advantage of a double-sampling plan \((n_1, c_1, n_2, c_2)\) is that it can have a similar OC curve to a single-sampling plan \((n, c)\) but with a smaller first sample: \(n_1 < n\).

So if a decision is reached at the first sample, less inspection is needed: \(n_1\) items, instead of \(n\).

But necessarily \(n_1 + n_2 > n\), so when the second sample is required (and 100% inspected), *more* inspection is needed.

Curtailing inspection in the second sample helps, but more inspection is still sometimes needed.
A graph of the Average Sample Number (ASN) against $p$ is helpful.

In R:

```r
source("R-code/AS.R")
plot(pd, ASmultiple(n = c(60, 120), c = c(1, 3), pd = pd)$ASN,
     type = "l")
lines(pd, ASmultiple(n = c(60, 120), c = c(1, 3), pd = pd,
                      curtail = c(FALSE, TRUE))$ASN, lty = 2)
abline(h = 89, lty = 3)
legend("topright",
       c("Complete inspection",
           "Curtailed inspection",
           "Single sampling"),
       lty = 1:3)
```

Compare the ASN curves with the constant 89 (Montgomery suggests that single sampling with $n = 89, c = 2$ has a similar OC curve).
Multiple-sampling plans

The extension to more than two stages of sampling is straightforward.

OC curves can be made using $OC_{2c}$, and ASN curves can be made using $AS_{multiple}$.

The aim of multiple-sampling, including double-sampling, is to achieve a desired OC curve with a low ASN.
Sequential-sampling plans

In *item-by-item sequential sampling*, the decision to:

- accept;
- reject;
- continue sampling;

is made after each item is inspected.

The acceptance and rejection numbers usually correspond to parallel sloping lines on the chart of (number nonconforming) versus (number inspected).

Inspection could continue indefinitely, but is usually terminated after a reasonable number of items have been inspected.
The parallel lines are found using Wald’s *sequential probability ratio test*.

For given PRP = \((p_1, 1 - \alpha)\) and CRP = \((p_2, \beta)\), the lines are

\[ X_A = -h_1 + sn \quad \text{(acceptance line)} \]

and

\[ X_R = h_2 + sn \quad \text{(rejection line)} \]

where \(h_1, h_2,\) and \(s\) are calculated from \(p_1, p_2, \alpha,\) and \(\beta\).

The OC curve for the resulting plan *approximately* respects the PRP and CRP.
The formulas are:

\[ h_1 = \left( \log \frac{1 - \alpha}{\beta} \right) / k \]

\[ h_2 = \left( \log \frac{1 - \beta}{\alpha} \right) / k \]

\[ k = \log \frac{p_2(1 - p_1)}{p_1(1 - p_2)} \]

\[ s = \log \left[ \frac{(1 - p_1)}{(1 - p_2)} \right] / k. \]
For example, if the PRP is \((p_1 = 0.01, 1 - \alpha = 0.95)\) and the CRP is \((p_2 = 0.06, \beta = 0.10)\), then

\[
X_A = -1.22 + 0.028n,
\]

\[
X_R = 1.57 + 0.028n.
\]

Note that \(X_A\) is negative for \(n < 1.22/0.028 \approx 44\), so at least 44 items must be inspected before the lot can be accepted.

Also, \(X_A < 1\) for \(n < 80\), so the acceptance number is effectively 0 for \(44 \leq n < 80\).
In R

PRP <- c(0.01, 0.95)
CRP <- c(0.06, 0.10)
plot(pd, ASsequential(PRP, CRP, 200, pd)$paccept, type = "l")
points(rbind(PRP, CRP))
plot(pd, ASmultiple(n = c(60, 120), c = c(1, 3), pd = pd,
    curtail = TRUE)$ASN,
     type = "l")
lines(pd, ASsequential(PRP, CRP, 200, pd)$ASN, lty = 2)