“The primary objective of time series analysis is to develop mathematical models that provide plausible descriptions for sample data...”

• We model a time series as a collection of random variables: $x_1, x_2, x_3, \ldots$, or more generally $\{x_t, t \in T\}$.

• Often the phenomenon being observed evolves in continuous time, but our observations are always discrete samples.
• If the sampling times $t_1, t_2, \ldots$ are equally spaced, their separation $\delta t = t_n - t_{n-1}$ is the sampling interval and $1/\delta t$ is the sampling rate (samples per unit time).

• Choice of sampling rate affects all aspects of data collection, analysis, and interpretation.
Example: White Noise

- Uncorrelated random variables $w_t$ with mean 0 and variance $\sigma_w^2$, written $w_t \sim \text{wn}(0, \sigma_w^2)$.

  - Why white noise?

  - By analogy with white light: in the frequency domain, all frequencies are present with the same same strength.

- If in addition the $w$’s are independent and identically distributed, we write $w_t \sim \text{iid}(0, \sigma_w^2)$. 
Iid White Noise

# t-distributed with 3 degrees of freedom:
w = ts(rt(500, df = 3));
plot(w);
If in addition the $w$’s are normally distributed, we write $w_t \sim \text{iid N}(0, \sigma_w^2)$. 
Iid Normal White Noise

\[ w = \text{ts(rnorm(500))}; \]
\[ \text{plot}(w); \]
Example: Moving Average

- Many observed series are smoother than white noise.

- Possible model:

\[ v_t = \frac{1}{3} (w_{t-1} + w_t + w_{t+1}) \]
Moving Average

\[ w = \text{ts(rnorm(500))}; \]
\[ v = \text{filter}(w, \text{sides} = 2, \text{rep}(1, 3) / 3); \]
\[ \text{plot}(v); \]
• Averaging attenuates the faster oscillations, leaving the slower oscillations more apparent.

• More generally, a *weighted* average of 2, 3, or more noise terms.
Example: Autoregression

- Recursive model:

\[ x_t = x_{t-1} - 0.9x_{t-2} + w_t, \quad t = 1, 2, \ldots, 500 \]

- Like a regression equation, but the RHS contains past ("lagged") LHS variables, hence autoregression.

- Shows many different types of behavior for different choices of coefficients.
Autoregression

\[ w = \text{ts(rnorm}(500)) \];
\[ v = \text{filter}(w, \text{filter} = c(1, -0.9), \text{method} = \text{"recursive"}) \];
\[ \text{plot}(v) \];
Example: Random Walk

- One model for trend; recursive definition:
  \[ x_t = \delta + x_{t-1} + w_t \]

- Explicitly:
  \[ x_t = \delta t + \sum_{j=1}^{t} w_j \]

- \( \delta \) is the *drift* (per unit time).
Random Walk

# drift delta = 0.2 per sample:
x = ts(cumsum(rnorm(500) + 0.2));
plot(x);
• The white noise we build it from could be non-normal.
Non-Normal Random Walk

# t-distributed increments, 1 degree of freedom, no drift:
x = ts(cumsum(rt(500, df = 1)));
plot(x);
Example: Signal in Noise

• Sine-wave signal:

\[ x_t = 2 \cos\left(\frac{2\pi t}{50} + 0.6\pi\right) + \omega_t, \quad t = 1, 2, \ldots, 500 \]

• More generally, the wave term could be

\[ A \cos(2\pi \omega t + \phi), \]

where:

– \( A \) is amplitude;

– \( \omega \) is frequency (in cycles per unit time);

– \( \phi \) is phase (in this case, in radians).
Cosine wave signal plus noise

\[ w = \text{ts(rnorm}(500)) ; \]
\[ x = 2 \times \cos(2 \times \pi \times \text{time}(w) / 50 + 0.6 \times \pi) + w ; \]
\[ \text{plot}(x); \]