Topic 2: The “Failures” of Risk Management

- Some news items:
  - An interview with the author of “The Black Swan.”

- Find these and others in the Newsletters of NCSU College of Management’s Enterprise Risk Management Initiative.
Value at Risk (VaR)

- A financial institution sets up many *positions*, each exposing it to the risk of a loss.

- Example: a coastal community issues bonds to raise $17 million to battle beach erosion.

- The issue is managed by an *underwriter*, which handles the legal work and finds investors to buy the bonds.

- The underwriter borrows the $17 million to pay to the community on the day the bonds are issued, but may take several days to complete the sale of the bonds to investors.
• So on day 0, the underwriter has:
  
  – a debt of $17 million to its lender;

  – an inventory of bonds initially worth $17 million;

  – a fee that it receives for its services.

• If the underwriter takes 5 days to find investors, and interest rates have changed in the interim, it may sell the bonds for more or less than the initial $17 million.

• That leads to a small but unpredictable profit or loss on the transaction.
• Write $V(t)$ for the net value of the positions on day $t$.

• Then $V(t) = V_1(t) + V_2(t)$, where:
  
  $V_1(t) = -$17 million, the obligation to repay the loan;

  $V_2(t) =$ market value of the bonds on day $t$.

• If the sale takes place after $h$ days, the profit or loss is $V(h) - V(0)$.
- The **Value at Risk** for a probability $\alpha$ and holding period $h$ days is the loss that is exceeded with probability $\alpha$:

$$P\{-[V(h) - V(0)] > \text{Var}_{\alpha,h}\} = \alpha.$$ 

- For example, $\text{Var}_{.05,5}$ is the loss that will be exceeded over a 5-day holding period with probability .05, or 5% of the time.

- Value at Risk is therefore just a quantile of the distribution of $V(h) - V(0)$. 
• When the transaction causes a loss, that is, the revenue from selling the bonds does not cover the loan, the underwriter must use some capital to make up the difference.

• If the loss exceeds the underwriter’s capital, it will default on the loan, and bad things result: bankruptcy, or a rescue by an affiliate, or ...

• So Value at Risk calculations can help to determine how much capital the underwriter must hold.

• All major banks calculate and report VaR; for example, Goldman Sachs’s latest quarterly report to the SEC.
The Distribution of Loss

- The key step in calculating VaR is getting the probability distribution of the potential loss, $-[V(h) - V(0)]$, or more generally $-[V(t + h) - V(t)]$.

- Since

$$V(t) = \sum_j V_j(t),$$

we need the joint distribution of $V_1(t), V_2(t), \ldots, V_N(t)$.

- The value of the $j^{th}$ position on day $t$, $V_j(t)$, is determined by the values of market variables like interest rates and exchange rates.
• Write $m_t$ for the vector of all such variables on day $t$.

• Then $V_j(t) = W_j(t, m_t)$ for some valuation function $W_j(\cdot, \cdot)$.

• Examples of valuation functions are:
  
  – the value of a bond as a function of interest rates;
  
  – the Black-Scholes formula for the value of an option as a function of interest rates and volatility.

• The randomness in $V_j(t)$ arises only from its dependence on $m_t$. 