ST 762, HOMEWORK 4 SOLUTIONS, FALL 2009

1. Results of my simulation with $S = 1000$ are as follows.

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SIMULATION RESULTS FROM  1000  MONTE CARLO DATA SETS

beta1 OLS
Bias = -1e-04 Rel Bias = 0 Rel Bias SD = -0.0288
Mean beta1,  SD beta1,  Mean estimated SE beta1,  and SD of estimated SEs,  Coverage
 2.2512018743 0.0031212093 0.0021301225 0.0005583524 0.7890000000

beta1 GLS-known
Bias = -1e-04 Rel Bias = 0 Rel Bias SD = -0.0182
Mean beta1,  SD beta1,  Mean estimated SE beta1,  and SD of estimated SEs,  Coverage
 2.2512348367 0.0023791748 0.0023332882 0.0004106543 0.9400000000

Efficiency of OLS relative to GLS theta known =  0.5808921
beta1 GLS-PL
Bias = -1e-04 Rel Bias = 0 Rel Bias SD = -0.0187
Mean beta1,  SD beta1,  Mean estimated SE beta1,  and SD of estimated SEs,  Coverage
 2.2512334134 0.0024650656 0.0023314571 0.0004920066 0.9300000000

Efficiency of OLS relative to GLS theta estimated =  0.6235832
Efficiency of GLS-estimated to GLS-known =  0.931539

beta2 OLS
Bias = 0 Rel Bias = 0 Rel Bias SD = -0.0066
Mean beta2,  SD beta2,  Mean estimated SE beta2,  and SD of estimated SEs,  Coverage
 -1.386329598  0.005313246  0.005557984  0.001456922  0.945000000

beta2 GLS-known
Bias = 0 Rel Bias = 0 Rel Bias SD = 0.0045
Mean beta2,  SD beta2,  Mean estimated SE beta2,  and SD of estimated SEs,  Coverage
 -1.3862703490  0.0016904116  0.0016263423  0.0002861369  0.9300000000

Efficiency of OLS relative to GLS theta known =  0.1012357
beta2 GLS-PL
Bias = 0 Rel Bias = 0 Rel Bias SD = 0.0057
Mean beta2,  SD beta2,  Mean estimated SE beta2,  and SD of estimated SEs,  Coverage
 -1.3862638519  0.0018283451  0.0014803774  0.0004125880  0.8640000000

Efficiency of OLS relative to GLS theta estimated =  0.1184400
Efficiency of GLS-estimated to GLS-known =  0.8547425
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(b) The biases, both raw and relative, are very small for both OLS and GLS.. This suggests that the consistency results translate into approximate unbiasedness in small samples.

(c) and (d) For both components of $\boldsymbol{\beta}$, the relative efficiency of OLS to GLS-known and GLS-PL is very much less than 1, suggesting that the precision of GLS methods relative to that of OLS is much greater. This is especially bad for β_2, with efficiency relative to the GLS methods in the range of 10-11% . The relative efficiency of OLS to GLS-known is somewhat less than that of OLS to GLS-PL, suggesting that having to estimate θ rather than knowing it incurs some penalty. Consequently, the relative efficiency of the GLS-PL estimator to the GLS-known estimator is less than 1, again highlighting that there is indeed a penalty for estimating θ (see (e) and (f) below for more).

(e) For OLS, the formula used to calculate the standard errors is incorrect, as it assumes constant variance. For β_1 the average of the standard error estimates is smaller than the Monte Carlo standard deviation of the 1000 estimates; for β_2, the average is slightly larger than and much closer to the Monte Carlo standard deviation. This latter result is probably just a matter of luck. Certainly the result for β_1 is consistent with the contention that failure to take account of the nonconstant variance can lead to unrealistically optimistic assessment of the precision of the OLS estimator.

For GLS-known, the average of estimated standard deviations is very close to the sample SD, suggesting that the theory is relevant. For GLS-PL, however, the average is a bit lower than the sample SD, suggesting that the "folklore" standard errors are a bit optimistic, particularly for β_2. This probably reflects the optimism discussed in the notes: in finite samples, there is an effect of estimating θ; it does not wash out as the folklore theory would suggest, leading to estimated standard errors that are a bit too small. This is discussed in greater detail in Chapter 11.

(f) The coverage probability for β_1 based on OLS is much smaller than 0.95, reflecting the fact that the standard errors used to calculate the intervals are much too small. The fact that the coverage is reasonable for β_2 is probably a matter of luck – the estimated standard errors for this parameter apparently are not too bad, leading to confidence intervals that do not understate the true coverage. For GLS-known, the coverages are close to 0.95, although that for β_1 is a bit too small; note that the standard error for estimating 0.95 based on a sample of size 1000 proportions is $\sqrt{(0.05)(0.95)/1000} = 0.0069$, which leads to a confidence interval for 0.95 of (0.936,0.964), which does not contain 0.93. This suggests that the theory may be a little optimistic even when θ is known at this finite sample size ($n = 18$). . For GLS-PL, the coverage for β_2 falls seriously short of the nominal level of 0.95. This again suggests that the folklore theory is optimistic.