1. Below is a vector $Y$ of responses to fertilizer treatments A, B, C, and D (in order) from a completely randomized design. I’ve also listed the data in a table to the right. You also see a design matrix $X$ having an intercept column and 3 other columns that I’ll call $X_1$, $X_2$, and $X_3$. The model is $Y_{ij} = \mu + \tau_i + e_{ij}$ for observation $j$ on fertilizer $i$.

\[
Y = \begin{pmatrix}
8 \\
6 \\
10 \\
13 \\
15 \\
14 \\
17 \\
19 \\
21 \\
11 \\
10 \\
12
\end{pmatrix}, \quad X = \begin{pmatrix}
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & -1 & -1 & -1 \\
1 & -1 & -1 & -1 \\
1 & -1 & -1 & -1
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Fertilizer</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fertilizer A</td>
<td>8</td>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Fertilizer B</td>
<td>13</td>
<td>15</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Fertilizer C</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Fertilizer D</td>
<td>11</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

(B) (15 pts.) Find the mean ___ for fertilizer A (the mean for fertilizer B is 14, for C it is 19 and for D it is 11).

Find $\sum_{j=1}^{3} (Y_{ij} - \bar{Y}_i)^2$ for $i=1$ ______ , $i=2$ ______ , $i=3$ ______ , and $i=4$ ______

(C) (24 pts.) Fill in the missing numbers in this analysis of variance table:

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>Source</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fertilizer</td>
<td>3</td>
<td>____________</td>
<td>___________</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>___</td>
<td>____________</td>
<td>__________</td>
<td>___</td>
</tr>
</tbody>
</table>

(D) (8 pts.) Compute the standard error _______ of $\bar{Y}_i - \frac{2}{3} \bar{Y}_2 - \frac{1}{3} \bar{Y}_3$.

(E) (12 pts.) List the regression coefficients you’d get when regressing the vector $Y$ on the matrix $X$. (hint: no need to compute $(X'X)^{-1}X'Y$ to do this)

$\hat{Y} = ___ + ___ X_1 + ___ X_2 + ___ X_3$

2. (18 pts.) An experiment on tree growth has growth measurements in feet. The standard error for the difference of two treatment means is 5 feet. If I change all of the data to inches (12 per foot) what will be the new standard error _______ of the difference of these two treatment means (means now in inches).
For the t test for the hypothesis that the true difference is 0, how much will the test statistic change?

3. (8 pts.) In a sample of 8 cars, we choose 4 at random to be equipped with a certain device and I want to know if it affects mileage (miles per gallon). The mean mileage for the 4 equipped cars was 0.503 higher than that of the 4 control cars. When I listed all possible ways of dividing the 8 cars into 2 groups of 4, there were 64 cases in which the absolute difference in average mileage between the two groups was less than .503. Compute the p-value ______ for a (two sided) test of the null hypothesis that the device has no effect on mileage versus the alternative that it changes the mileage.

4. Data from an incomplete block design with 2 fixed effect treatments $\tau_i$ and 5 random effect blocks $B_j$ gave data $Y_{ij}$ as shown here (x stands for a missing value). Also shown are formulas for weighted sample treatment means $\bar{Y}_i$.

$$
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & \text{weighted mean} \\
A & Y_{11} & Y_{12} & Y_{13} & x & Y_{15} \\
B & Y_{21} & Y_{22} & x & Y_{24} & Y_{25}
\end{array}
$$

$$(3Y_{11} + 3Y_{12} + Y_{13} + \ldots + 3Y_{15})/10 = \bar{Y}_1$$

$$(3Y_{21} + 3Y_{22} + \ldots + 3Y_{25})/10 = \bar{Y}_2$$

The model is $Y_{ij} = \mu + \tau_i + B_j + e_{ij}$ with $e_{ij} \sim N(0,1)$ and $B_j \sim N(0,8)$ with the usual independence assumptions for e and B. Notice that the variance components are given.

(A) (5 pts.) In terms of the model parameters, compute the expected value _______ of $\bar{Y}_1 - \bar{Y}_2$.

(B) (10 pts.) What is the variance _____ of this difference of weighted means $\bar{Y}_1 - \bar{Y}_2$?

**************answers **************

6, 8, and 10 average to 8 and the squares of 6-8, 8-8, and 10-1 sum to 4+0+4=8.
The other 3 sums of squares (trt B C and D) are similarly computed as
1+1+0=2, 4+0+4=8, and 0+1+1=10. Thus the error sum of squares is 8+2+8+2=20.
This has 2+2+2+2=8 df and the MSE is 20/8 = 2.5.
The treatment sum of squares using means 8, 14, 19 and 11 (overall mean 52/4=13) is
SS(trt) = 3(25+1+3 6+4) = 198 with 3 df 
3df + 8df = 11 df total.
MSq(trt) = 198/3 = 66 and F=66/2.5 = 26.4 (3 and 11 df)
The variance of each mean is 2.5/3 so the variance of the linear combination is
(2.5/3)(1+4/9+1/9) = 1.29 and the standard error is the square root of this, 1.14
This is the setup for the effects coding so we have intercept 13 (overall mean) and
parameters 8-13=-5, 1, and 6
13 -5X1+1X2+6X3

2. Each data point, mean, and contrast gets multiplied by 12 as does the standard error. Standard error becomes 12(5)=60 and t is unchanged (numerator and denominator both multiplied by 12).
3. There are \((8x7x6x5)/(4x3x2x1) = 7x2x5 = 70\) combinations with \(70-64 = 6\) greater than or equal to what we got (in magnitude) so p-value \(6/70\) randomization test.

4. We are estimating \(\tau_1 - \tau_2\) (\(\mu\) drops out and everything else is random)
Each Y has an e term in it, 6 of them are multiplied by \(3/10\) and 2 by \(1/10\), so that part of the variance is \([6(9/100) + 2(1/100)](1)\) where 1 is the e variance. Most of the block effects drop out but we are left with a difference of two of them, each multiplied by \(1/10\) so we have \(2(1/100)(8)\) where 8 is the block variance. This gives \(56/100 + 16/100 = 72/100 = 0.72\) as a variance.