Throughout, factors are denoted A, B, C, etc. as usual.

1. (14 pts.) How many equal signs ______ would appear in the defining contrast for an unreplicated 2^{8-3} fractional factorial experiment (one eighth of a 2^8)? How many observations _______ would the experiment have?

2. (18 pts.) I ran a central composite design and analyzed it with a full quadratic response surface in PROC RSREG. The output showed an F test for interactions $F_{20}^{6} = 3.61$ with p-value 0.0138.

   (a) At the usual $\alpha=0.05$ level, do I decide there are or are not interactions in my model?

   (b) Part (a) tells me what to do with my 0.0138 p-value, but what exactly is that p-value, that is, what is the computer computing when it gets 0.0138? You could label a graph of F as your answer if you like.

   (c) If I had four factors in this experiment, how many interactions _______ would I have had? Can you tell from the given information above how many factors were in this experiment? If so, how can you tell?

3. (24 pts.) I have a one eighth replicate of a 2^{12} experiment (2^{12-3}). It has resolution five. I fit a model with all main effects and all two way interactions getting error sum of squares 8000. The sum of all responses with factor A at the high level was 240 and the sum for A at the low level was 160. The A main effect was the first thing entered in my model. Compute if possible

   (a) The error mean square __________

   (b) The Type I (sequential) sum of squares for A __________

   (c) The Type III (partial) sum of squares for A __________

   (d) Would your answer for Type III in part (c) change if the resolution had been three with A aliased with the BC interaction? (yes, no) If yes, explain briefly the change.
4. (10 pts.) Suppose you ran an unreplicated $2^4$ design, fitting main effects and all possible interactions. Would you use hypothesis testing to find which effects are important? (yes, no) If no, what technique would you use? If yes, what alpha level would you use?

5. (8 pts.) I will run an experiment with all 25 combinations of $X_1 = 0, 0.05, 0.10, 0.15, 0.20$ and $X_2 = 0, 0.05, 0.10, 0.15, 0.20$ where $X_1$ is the proportion of drug A, $X_2$ the proportion of drug B, and $X_3 = 1-X_1-X_2$ the proportion of water in a drug mixture. The response $Y$ is effectiveness of the drug mixture. I will run each combination once (25 runs). I plan to find the most effective proportions by regressing $Y$ on $X_1, X_2, X_3, X_1^2, X_2^2, X_1X_2, X_1X_3, X_2X_3$ with an intercept. How many error degrees of freedom ____ will show up on my printout when I run this regression?

6. (10 pts.) Here is part of the defining contrast for a one eighth replicate of a $2^{15}$ experiment ($2^{15-3}$).

$$I = ABCDE=CDEFG=\text{______________________________}$$

If possible, fill out the rest of this defining contrast in such a way that the resulting design has resolution five. If not possible, explain briefly why it can’t be done.

7. (16 pts.) I have a $2^{8-2}$ (quarter replicate of a $2^8$) experiment in which interaction ACH is aliased with (at least) two other interactions as follows:

$$ACH = -CDE=EF GH$$

(a) If possible, write out the defining contrast $I = \text{______________________________}$

(b) Is anything else aliased with ACH besides what is shown here? (yes, no) If so, what is it?
1. 32 observations and 8 effects (if you include I) and thus 7 = signs in the defining contrast.

2. There are interactions (p-value<0.05). Plot computed F on horizontal axis and label area to right as p-value or “this is the probability of getting F>3.61 when there are no interactions.”
   4x3/2=6 interactions which would also be numerator df for F test for interactions (so this one had 4 factors)

3. n=512 observations, 12 main effects and 12(11)/2=66 interactions (2-way) and 1 intercept
   512-1-78=433 error df, MSE = 8000/433=18.48. With resolution five, all effects are estimable and these plans have orthogonal design matrices so SS(A) is same for type I and III and it is (240-160)²/512=80²/512=12.5. If resolution were three, we’d have aliasing so Type III SS for A would be 0 as Type III assumes that factor enters last.

4. Lenth or Daniels – you have no MSE to do t test.

5. Coefficients are not uniquely estimable. This also means error df will not be 25 -1 -9 but instead will be what I’d get if I left out all the X3 terms so 25-1-5 = 19.

6. Multiply ACH = -CDE=EFGH by ACH to get I=-ADEH=ACEFGH and include generalized interaction
   -CDFGH which also shows that ACH is aliased with ADFG (ACH=-ADFG).