Marginal and Conditional

Define Y_1 = number of dots on the upper face of die 1, and Y_2 is the number of dots on the upper face of die 2.

For all distinct paris of values y_1, y_2 the bivariate events $(Y_1 = y_1, Y_2 = y_2)$ represented by (y_1, y_2) are mutually exclusive events. Then, the univariate event $(Y_1 = y_1)$ is the union of bivariate events of the type $(Y_1 = y_1, Y_2 = y_2)$ with the union being taken over all possible values for y_2. Then,

$$p(Y_1 = y_1) = p_1(y_1) = \sum_{y_2} p(y_1, y_2)$$

when $y_1 = 1, 2, \ldots, 6$, it is the marginal probability function for Y_1.

Definition

- Let Y_1 and Y_2 be jointly discrete r.v. with probability function $p(y_1, y_2)$. Then the *marginal* probability functions of Y_1 and Y_2 are given by

$$p_1(y_1) = \sum_{y_2} p(y_1, y_2)$$

and

$$p_2(y_2) = \sum_{y_1} p(y_1, y_2)$$

- Let Y_1 and Y_2 be jointly continuous r.v. with joint density function $f(y_1, y_2)$. Then the *marginal density* functions of Y_1 and Y_2 are given by

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2$$

and

$$f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

Definition

If Y_1 and Y_2 are jointly discrete r.v. with joint probability function $p(y_1, y_2)$ and marginal probability functions $p_1(y_1)$ and $p_2(y2)$ respectively, then the *conditional discrete probability function* of Y_1 given Y_2 is

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(}{p}$$

provided that $p_2(y_2) > 0$.

Definition

If Y_1 and Y_2 are jointly *continuous* r.v. with joint density function $f(y_1, y_2)$ then the *conditional distribution function* of Y_1 given $Y_2 = y_2$ is

$$F(y_1|y_2) = P(Y_1 \leq y_1|Y_2 = y_2)$$

Definition

Let Y_1 and Y_2 be jointly continuous r.v. with joint density $f(y_1, y_2)$ and marginal densities $f_1(y_1)$ and $f_2(y_2)$ respectively. For any y_2 such that $f_2(y_2) > 0$ the conditional density of Y_1 given $Y_2 = y_2$ is given by

$$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$

and for any $y_1$such that $f_1(y_1) > 0$ the conditional density of Y_2 given $Y_1 = y_1$ is given by

$$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$