Probability

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Interpretation

Probability is a measure of one's belief in the occurrence of a future event. A prabability is a value between 0 and 1.

Example: if the probability of rain is .5, that means that there is a 50% chance that we will get rain. If the probability of rain is 1, that means it will rain for sure. If the probability is 0, then it will not rain (without doubt!).

The relative frequency is a meaningful measure of our belief in the occurrence of an event.

Example: the relative frequency of a head in a long series of a coin tosses, is an approximation to the probability of head but it does not provide a rigorous definition of probability.

Probability and Inference

We need a theory of probability that will pemit us to calculate the probability of observing specified outcomes, assuming that our hypothesized model is correct.

Example presented in class.

Set Notation

Notation:

We use capital letters, $A, B, C, \ldots$ to denote sets of points

If the elements in the set A are a_1, a_2, and a_3, we will write

$$A = \{a_1, a_2, a_3\}$$

A is contained in B: $A \subset B$

Null set ϕ

S is the universal set (with all the elements under consideration).

$A \cup B$, A union B

$A \cap B$, A intersection B

$\bar{A}$, complement of A.

If $A \cap B = \phi$ then A and B are disjoint or mutually exclusive.

Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's laws

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

Probabilistic Model

An experiment is the process by which an observation is made.

Examples: coin and die tossing, measuring the IQ score of an individual.

The outcomes of an experiment are called events.

An event which can be decomposed into other events, is called a compound event. An event that cannot be decomposed is called simple event.

Sets are collections of points. Then, we associate a distinct point, called a sample point, with each and every simple event associated with an experiment.

A simple event is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point. The letter E with a subscript will be used to denote a simple event or the corresponding sample point.

The sample space associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S.

A discrete sample space is one that contains either a finite or a countable number of distinct sample points.

An event in a discrete sample space S is a collection of sample points that is, any subset of S.

Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S) we assign a number, $P(A)$, called the probability of A, so the following axioms hold:

- Axiom 1: $P(A) \geq 0$.
- Axiom 2: $P(S) = 1$.
- Axiom 3: If $A_1, A_2, A_3, \ldots$ form a sequence of pariwise mutually exclusive events in S (that is, $A_i \cap A_j = \phi$ if $i \neq j$), then

$$P(A_1 \cup A_2 \cup A_3 \cdots) = \sum_{i=1}^{\infty} P(A_i).$$