

## Theorem

$$\begin{aligned} E(cg(Y)) &= \sum_y cg(y)p(y) = c \sum_y g(y)p(y) \\ &= cE(g(Y)) \end{aligned}$$

**Theorem** Let  $Y$  be a discrete r.v. with probability function  $p(y)$  and  $g_1(Y), g_2(Y), \dots, g_k(Y)$  be  $K$  functions of  $Y$ . Then,

$$\begin{aligned} &E[g_1(Y) + g_2(Y) + \dots + g_k(Y)] \\ &= E[g_1(Y)] + E[g_2(Y)] + \dots + E[g_k(Y)] \end{aligned}$$

**Theorem** Let  $Y$  be a discrete r.v. with probability function  $p(y)$ ; then

$$\sigma^2 = V(Y) = E[(Y - \mu)^2] = E(Y^2) - \mu^2.$$

## The binomial

In this section we are concerned with experiments, known as *binomial* experiments, that exhibit the following characteristics.

A binomial experiment possesses the following properties:

1. The experiment consists of  $n$  identical trials.
2. Each trial results in one of two outcomes. We will call one outcome a success  $S$  and the other a failure  $F$ .
3. The probability of success on a single trial is equal to  $p$  and remains the same from trial to trial. The probability of a failure is equal to  $q = (1 - p)$
4. The trials are independent.
5. The random variable of interest is  $Y$ , the number of successes observed during the  $n$  trials.

Notice that the random variable of interest is the number of successes observed in the  $n$  trials. A success is not necessarily “good” in the everyday sense of the word.

Example:  $Y =$  total number of heads when we toss 2 coins.

## Definition

A random variable  $Y$  is said to have a *binomial distribution* based on  $n$  trials with success probability  $p$  if and only if

$$p(y) = \binom{n}{y} p^y q^{n-y}$$

for  $y = 0, 1, 2, \dots, n$ , and  $0 \leq p \leq 1$ .

The binomial probability distribution has many applications because the binomial experiment occurs in sampling for defectives in industrial quality control, in the sampling of consumer preference or voting populations, and in many other physical situations.

## **Theorem**

### MEAN AND VARIANCE FOR THE BINOMIAL:

Let  $Y$  be a binomial random variable based on  $n$  trials and success probability  $p$ . Then,

$$\mu = E(Y) = np$$

and

$$\sigma^2 = V(Y) = npq$$