

**ST 778, Fall 2007**  
**Measure Theory and Advanced Probability I**  
**<http://www.stat.ncsu.edu/people/fuentes/st778>**

**Instructor:**

Dr. Montserrat Fuentes  
Office: Patterson 209 G , phone: 515-1921  
email: [fuentes@stat.ncsu.edu](mailto:fuentes@stat.ncsu.edu)  
webpage: [www4.stat.ncsu.edu/~fuentes](http://www4.stat.ncsu.edu/~fuentes)  
Office Hours: Tuesday 12-1pm

**TA:**

Jiangtao Duan  
email: [jduan@ncsu.edu](mailto:jduan@ncsu.edu)  
Office hours: Mondays from 3:20 to 4:20pm  
and Wednesdays from 3:20 to 4:20pm  
Location: room #9 Patterson Hall

**Course prerequisites:**

MA 426, ST 512 or MA(ST) 546 or equivalent.

**Required Textbook:**

Probability and Measure. Patrick Billingsley. Third Edition. Wiley.  
(The book is available in NCSU Central Bookstore.)

**Schedule:**

Patterson #208: 1:30-2:45PM TU. and TH.

**HomeWork:**

HW's will usually be assigned every Thursday. You will get a week time to turn in the HW's.  
Copying is prohibited though you may talk to each.

**Exams:**

There will be one midterm exam and the final exam. The midterm exam will take place October 16 from 1:30 to 2:45pm. **Final exam** will take place December 11 from 1 to 4pm. Both exams will be closed book, closed notes. You may bring two pages of formula sheets. Any topic discussed in the class prior to the exam will be fair game. Final exam will be comprehensive covering everything discussed in the course.

## Grading policy:

The course grade will be based on HW's and exam. The relative weight given to each of these components is

HW's: 10%, Midterm: 40%, Final exam.: 50%.

Scores will be converted to letter grades. +/- grades will be assigned for each grade level.

## Objectives:

The course will follow the text closely. My goal is to keep the lectures fresh and interesting but still draw substantially from Billingsley.

Each of the topics listed below is roughly the material that will be covered in one or two lectures. Moreover the pace will be slow in the first few weeks and then increase as we become familiar with notation and the standard techniques used in measure theory proofs. The sequencing follows the text but we will skip many of the more detailed examples and will rarely make excursions into the starred sections. For this first pass through modern, abstract integration is as important to focus on the core of the theory and not to get confused by tangential issues. The first few lectures are actually a mini-course in probability theory to motivate the measure theory. I am skipping Chapter 1 completely. This is good Summer reading or something to look over during the Christmas recess.

- Probability Introduction
  - Sigma-fields
  - Probability measures
  - Statement of extension theorem for probability measures
  - Infinite sequences
  - Independence and the Borel-Cantelli lemmas.
  - Simple random variables.
  - Statement of the strong law of large numbers
- Measure theory
  - General measures
  - Outer measure and the extension theorem
  - Lebesgue measure
  - Measurable functions
  - Distribution functions
- The integral for real valued functions
  - Definition and properties
  - Equivalence to Riemann integral
  - monotone convergence theorem
  - Dominated convergence theorem
  - Change of variables

- Uniform integrability
- Change of variables for Lebesgue densities
- Product Measures and conditional expectation
  - Product spaces and product measures
  - Fubini's Theorem
  - Conditional expectation and basic properties
  - Conditional probability
- Random variables
  - Convergence in probability
  - Independence
- Expectation as an integral
  - moments
  - Inequalities
  - Independence
- $L_p$  Spaces

Online class evaluations will be available for students to complete during the last two weeks of class (November 26-December 9). Students will receive an email message directing them to a website where they can login using their Unity ID and complete evaluations. All evaluations are confidential; instructors will never know how any one student responded to any question, and students will never know the ratings for any particular instructors.