

**Homework #1 ST 778 Fall 2007**  
**Dr. Fuentes**

- (1) Show by DeMorgan's law

$$(\limsup A_n)^c = \liminf A_n^c \quad \text{and}$$

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- (2) If  $A_n \rightarrow A$  and  $B_n \rightarrow B$  show that  $A_n \cup B_n \rightarrow A \cup B$   
and  $A_n \cap B_n \rightarrow A \cap B$ .

- (3) If  $A_n \rightarrow A$  show that  $A_n^c \rightarrow A^c$ .

- (4) Show that  $\limsup (A_n \cup B_n) = \limsup A_n \cup \limsup B_n$   
whereas  $\limsup (A_n \cap B_n) \subset \limsup A_n \cap \limsup B_n$   
Give an example to show that it is a proper inclusion.

- (5) Show that  $\liminf (A_n \cup B_n) \supset \liminf A_n \cup \liminf B_n$ .  
 $\liminf (A_n \cap B_n) = \liminf A_n \cap \liminf B_n$ .

- (6) Show that  $\limsup A_n - \liminf A_n = \limsup (A_n A_{n+1}^c)$   
 $= \limsup (A_n^c A_{n+1})$