Ch.1-Ex.12: Using Example 1.4, we know that $\Pr[X = 3|\lambda] = \frac{\lambda^3 e^{-\lambda}}{3!}$. As the $\lambda$ that maximizes $\Pr[X = 3|\lambda]$, also maximizes $\log \Pr[X = 3|\lambda] = 3 \log(\lambda) - \lambda - \log(3!)$ we find $\lambda$ by solving
\[
\frac{d \log \Pr[X = 1|\lambda]}{d \lambda} = \frac{3}{\lambda} - 1 = 0
\]
Clearly $\lambda = 3$ solves the above equation. Also $\frac{d^2 \log \Pr[X = 3|\lambda]}{d \lambda^2} = -\frac{3}{\lambda^2} < 0$ implies that $\lambda = 3$ maximizes $\Pr[X = 3|\lambda]$.

Ch.1-Ex.16: Since the spinner is unbiased, it can land at any random value between 0 and 1 and hence the pdf is given by $p(y) = I(0 < y \leq 1)$.

Recall that the indicator function, $I(A) = 1$ if $A$ is true, otherwise $I(A) = 0$.

Ch.1-Ex.25: Given that the three bases $(xxx)$ are independent and mutate with the same probability $p$ over the course of a life time, the distribution of the number of mutations, $X \sim Bin(N = 3, \theta = p)$ (see p.15 of the textbook). Thus, it follows (see p. 33 and 35) that the mean and variance are given by $E(X) = N\theta = 3p$ and $Var[X] = N\theta(1 - \theta) = 3p(1 - p)$.

Ch.1-Ex.26: Given the correctness of one sentence does not affect the correctness of any other sentence and every sentence has a 90% of being grammatically correct, the total number of correct sentences, $X \sim Bin(N = 10, \theta = 0.9)$ (see p.15 of the textbook). The probability that there are no more than 2 incorrect sentences out of 10 is $\Pr[X \geq 8] = 1 - \Pr[X \leq 7] = \sum_{k=8}^{10} \binom{10}{k}(0.9)^k(1 - 0.9)^{10-k} = 0.9298$.

R code:
\[
> \text{dbinom}(8,10,0.9)+\text{dbinom}(9,10,0.9)+\text{dbinom}(10,10,0.9)
\]
Or
\[
> 1-\text{pbinom}(7,10,0.9)
\]

Ch.1-Ex.27: Given that $\Pr(A \text{ wins a game}) = 0.6$, it follows that $\Pr(B \text{ wins a game}) = 1 - 0.6 = 0.4$. Notice that B wins the series in of the following cases:
(i) B wins all 4 out of first 4 games, or
(ii) B wins 4 out of first 5 games, or
(iii) B wins 4 out of first 6 games, or
(iv) B wins 4 out of first 7 games.

Now we can compute the probability of the events (i)-(iv). Notice that
\[ \Pr[(i)] = (0.4)^4, \]
\[ \Pr[(ii)] = \binom{4}{1} (0.6)(0.4)^4, \]
\[ \Pr[(iii)] = \binom{5}{2} (0.6)^2(0.4)^4, \]
\[ \Pr[(iv)] = \binom{6}{3} (0.6)^3(0.4)^4. \]

Hence \( \Pr[ B \text{ wins the series } ] = \sum_{k=0}^{3} \binom{k+3}{k} (0.6)^k(0.4)^4 = 0.2898. \)

\( R \) code:
\[
> k=0:3 \\
> \text{sum( choose(k+3,k)*(0.6)^k*(0.4)^4 )}
\]

Ch.1-Ex.28:  (a) Since the 10 shots are independent and the chance of making a shot is 0.7, it follows that the number of shots she makes is \( m \sim \text{Bin}(N = 10, \theta = 0.7). \) Now from p.33 and p.35 it follows that the mean and variance of a Binomial distribution is given by
\[ \mathbb{E}(m) = N\theta = 10*0.7 = 7 \quad \text{and} \quad \text{Var}(m) = N\theta(1-\theta) = 10*0.7*(1-0.7) = 2.1. \]
Also the probability she makes between 5 and 9 shots is \( \Pr[5 \leq m \leq 9] = \sum_{k=5}^{9} \binom{10}{k} (0.7)^k(0.3)^{10-k} = 0.9244. \)

\( R \) code:
\[
> \text{N=10;theta=0.7} \\
> \text{N*theta} \\
> \text{N*theta*(1-theta)} \\
> \text{sum(dbinom(5:9,N,theta))}
\]

(b) The only way her team gets a rebound is that she misses her shot. So \( \Pr[\text{rebound}] = \Pr[\text{she misses}] \times \Pr[\text{her team gets rebound}] = 0.3 \times 0.3 = 0.09. \) As there can a total of 10 rebounds, it follows that \( r \sim \text{Bin}(N = 10, \theta = 0.09). \) Hence it follows that the mean and variance are given by
\[ \mathbb{E}(m) = 10 \times 0.09 = 0.9 \quad \text{and} \quad \text{Var}(m) = 10 \times 0.09 \times (1 - 0.09) = 0.819. \] Also \( \Pr(r \geq 1) = 1 - \Pr(r = 0) = 1 - 0.91^{10} = 0.6106. \)
Ch.1-Ex.36: (a) Notice that the answer could be "yes" from either question (a) or question (b). Hence

\[
\Pr[\text{answer is yes}] = \Pr(\text{answer is yes}|\text{question a}) \Pr(\text{question a}) + \Pr(\text{answer is yes}|\text{question b}) \Pr(\text{question b})
\]

\[
= \frac{1}{2} \cdot \frac{1}{2} + p \cdot \frac{1}{2}
\]

\[
= \frac{1}{2^p + \frac{1}{4}} = \frac{2p + 1}{4}
\]

(b) Since all of the 100 randomly chosen persons’ answers are independent, so the number who answer yes is \(X \sim \text{Bin}(N = 100, \theta = \frac{2p+1}{4})\).