Sample Questions for Exam III
ST422 - Introduction to Mathematical Statistics-II
The ACTUAL exam will consists of less number of problems.

1. Let $Y_1, Y_2,\ldots,Y_n$ be a random sample from some density which has mean $\mu$ and variance $\sigma^2$.

   (a) Show that $\sum_{i=1}^{n} a_i X_i$ is an unbiased estimator of $\mu$ for any set of known constants $a_1, a_2,\ldots,a_n$ satisfying $\sum_{i=1}^{n} a_i = 1$.

   (b) If $\sum_{i=1}^{n} a_i = 1$, show that $Var[\sum_{i=1}^{n} a_i Y_i]$ is minimized for $a_i = \frac{1}{n}$, $i = 1,\ldots,n$.

   What does this result say about the relative efficiency of $\bar{Y}$?

2. Find a 90% C.I. for the mean of a normal distribution with $\sigma = 3$ given the sample $(3.3, -0.3, -0.6, -0.9)$. What would be the confidence interval if $\sigma$ were unknown?

3. Develop a method for estimating the ratio of variances of two normal populations by a confidence interval.

4. Let $Y$ be a single observation from a $Beta(\theta, 1)$ distribution, where $\theta > 0$.

   (a) Find a pivotal quantity.

   (b) Use the pivotal quantity of (a) to find a 95% C.I. of $\theta$.

5. Let $Y_1,\ldots,Y_n \sim_{i.i.d} N(\theta, \theta^2)$, $\theta > 0$.

   (a) Find a sufficient statistic for $\theta$.

   (b) Show that $\bar{Y}^2$ is a biased estimate of $\theta^2$.

   (c) Find a number $c$ such that $c\bar{Y}^2$ is an unbiased estimate of $\theta^2$.

   (d) Find a consistent estimate of $\theta$.

   (e) Find a pivotal quantity of $\theta$.

   (f) Use the pivotal quantity of (e) to find a 90% C.I. of $\theta$. 
6. Let $Y_1, Y_2, \ldots, Y_n \sim \text{i.i.d. } Exp(\theta)$.

   (a) Show that $\bar{Y}$ is a sufficient statistic for $\theta$.

   (b) Suggest an unbiased estimator for $\theta$, and show how would you estimate the standard error of your estimator.

   (c) Is your estimator in (b) consistent for $\theta$?

   (d) Show that $2n\bar{Y}/\theta$ is a pivotal quantity.

   (e) Use the above pivotal quantity in (d) to obtain a $100(1 - \alpha)$% C.I. for $\theta$.

7. Solve the **suggested problems from the text** (see lecture slides).

8. Additional suggested problems from the text: 8.6, 8.21, 8.24, 8.63, 8.91, 8.94, 9.40. *(I strongly recommend you to solve all these additional problems)*

   Solutions to above problems will NOT be posted on the web. However, I’ll solve the problems in class.