1. Prove that if $P(A) > 0$ and $P(B) > 0$, then
   (i) If $A$ and $B$ are mutually exclusive, they cannot be independent.
   (ii) If $A$ and $B$ are independent, they cannot be mutually exclusive. [20]

2. Two people each toss a fair coin $n$ times. Find the probability that they will toss the same number of heads. [10]

3. In answering a question on a multiple-choice test, a student either (correctly) knows the answer or guesses. Let $p$ be the probability that the student knows the answer and $1 - p$ be the probability that the student guesses. Assume that a student who guesses at the answer guesses completely at random, that is the student will be correct with probability $1/m$, where $m$ is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer, given that he or she answered it correctly? [20]

4. Let $X$ be a continuous random variable with the density function

   $$f_X(x) = \begin{cases} \frac{1}{2}x^2, & \text{if } -2 \leq x \leq 1 \\ 0, & \text{o.w.}\end{cases}$$

Define $Y = X^2$. Find the density function of $Y$. [20]

5. The random variable $X$ have the density

   $$f_X(x) = xe^{-x}, \quad x > 0.$$ 

Find the density of $Y = X^3$, and compute the expected value $E(Y)$? [10]

6. Let $X$ be a discrete random variable taking values $0, 1, 2, \ldots$ with probabilities

   $P(X = 0) = 1 - \alpha + \alpha p$ and for $k = 1, 2, \ldots$, $P(X = k) = \alpha pq^k$, where $q = 1 - p$, $0 < p < 1$, $0 < \alpha < 1$.

(i) Find the moment generating function of $X$.
   (ii) Compute the $E(X)$ and $Var(X)$ based on the moment generating function obtained from (i). [20]