1. Let $X \sim \Gamma(n, \beta)$.

(a) Show that
$$\lim_{n \to \infty} \Pr\left( \frac{X - n\beta}{\sqrt{n\beta^2}} \leq t \right) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du.$$ 

(b) Show that $\sqrt{2X} - \sqrt{2n\beta} \to_d N(0,1)$ as $n \to \infty$.

2. Let $X_1, \ldots, X_n$ be $n$ independent observations from the density
$$f(x) = \frac{1}{2} x^2 e^{-x}, \text{ for } x > 0.$$ 

Let $T_n = \frac{1}{n} \sum_{i=1}^{n} X_i$.

(a) Find the constant $\theta$ such that $T \to^p \theta$.

(b) Is it true that $T \to^{a.s.} \theta$? Justify your answer.

(c) Find constants $a$ and $b$ such that $\sqrt{n}(T_n - a)/b$ converges in distribution to $N(0,1)$ as $n \to \infty$. 