1. A point estimator is a numerical value which is considered to be a reasonable good guess of the true population parameter.
   (a) True
   (b) False

2. Suppose we want to estimate an unknown parameter $\theta$. Choose the correct answer.
   (a) An estimator $\hat{\Theta}$ is called unbiased if $E(\hat{\Theta}) = \theta$
   (b) An estimator $\hat{\Theta}$ is called unbiased if $E(\hat{\Theta}) \neq \theta$
   (c) $\hat{\Theta}$ has the smallest variance among all the estimators.
   (d) None of the above.

3. Suppose $X_1, \ldots, X_n$ is a random sample from a distribution with mean $\mu$ and variance $\sigma^2$. Which of the following is true?
   (a) Sample mean is an unbiased estimator for $\mu$
   (b) Sample variance is an unbiased estimator for $\sigma^2$
   (c) Both (a) and (b)
   (d) None of the above

4. Suppose $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$. The sampling distribution of the sample mean $\bar{X}$ is

5. Suppose $\hat{\Theta}$ is MLE of $\theta$. Which of the following is true:
   (a) $\hat{\Theta}$ is also MOM estimator of $\theta$
   (b) $3\hat{\Theta}$ is MLE of $3\theta$
   (c) Both (a) and (b)
   (d) None of the above
6. The following sample of size \( n = 5 \) was drawn from a population with mean \( \mu \) and standard deviation \( \sigma \): 8.9, 7.1, 5.8, 11.2, 9.2.

   (a) Calculate point estimates of the population mean and standard deviation.
   (b) What is the estimated standard error of the estimator you used for \( \mu \)?

7. Discuss how each of the following factors affects the width of the large-sample confidence interval for \( \mu \) when \( \sigma \) is known.

   (a) Confidence coefficient \((1 - \alpha)\).
   (b) Sample size \((n)\).
   (c) Population standard deviation \((\sigma)\).

8. Let \( X_1, \cdots, X_n \) be a random sample from a distribution with density

   \[ f(x; \theta) = (\theta - 1)x^{-\theta}, \quad x > 1, \theta > 2. \]

   (a) Find \( E(X) \)
   (b) Show that the MOM estimator of \( \theta \) is \((1 - 2\bar{X})/(1 - \bar{X})\).
   (c) Show that the MLE of \( \theta \) is \(1 + n/\sum_{i=1}^{n} \ln(X_i)\).
   (d) Find the MLE of \( 3\theta + 1 \).
   (e) Suppose we observe the following 5 observations: 1.3, 1.2, 8.6, 5.2, 1.2. Calculate the method of moments estimate of \( \theta \).
   (f) Based on the same data, calculate the maximum likelihood estimates of \( \theta \) and \( 3\theta + 1 \), respectively.
9. Suppose that the lengths of the trout fry in a pond at the fish hatchery have the overall standard deviation of 0.8 inch. A random sample of 49 fry will be netted and their lengths measured. Answer the following questions. You may use the following information:

\[ z_{0.025} = 1.96, \quad z_{0.05} = 1.645, \quad z_{0.03} = 1.88 \]

(a) What is the probability that the sample mean will be within 0.2 inch of the population mean \( \mu \)?

(b) Suppose the sample mean of the 49 fry netted is \( \bar{x} = 3.4 \) inches. Construct a 95% confidence interval for the overall mean length of the trout fry in the pond.

(c) What is the minimum sample size required obtaining a 95% confidence interval with width 0.4 inches?

(d) Construct a 90% confidence interval for the overall mean length of the trout fry in the pond.

10. A television station wants to estimate the proportion of the viewing audience in its area that watch its evening news. 105 people were surveyed and only 30 of them watch the evening news regularly. Answer the following questions. You may use the following information:

\[ z_{0.025} = 1.96, \quad z_{0.05} = 1.645, \quad z_{0.03} = 1.88 \]

(a) Construct a large sample 95% confidence interval for \( p \), the true proportion of the viewing audience in its area that watch its evening news.

(b) Find the minimum sample size required to obtain a 95% confidence interval with width 0.06.

11. Let \( X_1, \ldots, X_{50} \) be a random sample from a distribution which has mean \( \mu \) and variance \( \sigma^2 \). Also suppose that \( a_1, \ldots, a_{50} \) are known numbers (not random variables).

(a) Compute \( E(\sum_{i=1}^{50} a_i X_i) \)

(b) Compute \( V(\sum_{i=1}^{50} a_i X_i) \)