Problems

1. Consider a normal population distribution with the value of $\sigma$ known.
   a) What is the confidence level for the interval $\bar{x} \pm 2.81 \frac{\sigma}{\sqrt{n}}$?
   b) What is the confidence level for the interval $\bar{x} \pm 1.44 \frac{\sigma}{\sqrt{n}}$?
   c) What value of $z_{a/2}$ in the CI formula: $\bar{x} \pm z_{a/2} \frac{\sigma}{\sqrt{n}}$ results in a confidence level of 99.7%?
   d) Answer the question posed in part (c) for a confidence level of 75%.

2. Each of the following is a confidence interval for $\mu =$ true average (i.e., population mean) resonance frequency (Hz) for all tennis rackets of a certain type. They were calculated from the same sample data, using two different confidence levels, and assuming $\sigma^2$ known:

   (114.4, 115.6) (114.1, 115.9)

   a) What is the value of the sample mean resonance frequency?
   b) The confidence level for one of these intervals is 90% and for the other is 99%. Which of the intervals has the 90% confidence level, and why?
   c) Which interval is more precise?

3. Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and the alcohol content of each bottle is determined. Let $\mu$ denote the average alcohol content for the population of all bottles of the brand under study. Suppose that the resulting 95% confidence interval is (7.8, 9.4).
   a) Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.
   b) Consider the following statement: There is a 95% chance that $\mu$ is between 7.8 and 9.4. Is this statement correct? Why or why not?
   c) Consider the following statement: We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4. Is this statement correct? Why or why not?

4. A CI is desired for the true average stray-load loss $\mu$ (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed with $\sigma = 3.0$.
   a) Compute a 95% CI for $\mu$ when $n = 25$ and $\bar{x} = 58.3$. Express this CI with a sentence.
   b) Compute a 95% CI for $\mu$ when $n = 100$ and $\bar{x} = 58.3$. Now I just want you to see how the CI changes when different quantities change, so don’t bother writing the sentence.
c) Compute a 99% CI for µ when n = 25 and \( \bar{x} = 58.3 \).

d) Compute an 82% CI for µ when n = 25 and \( \bar{x} = 58.3 \).

e) How large must n be if the width of the 99% interval for µ is to be 1.0?

5. By how much must the sample size n be increased if the width of the CI is to be halved? If the sample size is increased by a factor of 25, what effect will this have on the width of the interval? Justify your assertions.

6. Consider the next 1000 confidence intervals for µ at the 95% level that a statistical consultant will obtain for various clients. Suppose the data sets on which the intervals are based are selected independently of one another.

   a. How many of these 1000 intervals do you expect to capture the corresponding value of µ?

7. Determine the confidence level for each of the following large-sample one-sided confidence bounds:

   a) Upper bound: \( \bar{x} + 0.84 \frac{\sigma}{\sqrt{n}} \)
   
   b) Lower bound: \( \bar{x} - 2.05 \frac{\sigma}{\sqrt{n}} \)
   
   c) Upper bound: \( \bar{x} + 0.67 \frac{\sigma}{\sqrt{n}} \)

8. This is taken from Chapter 1, problem 36 of your text. A sample of 26 offshore workers took part in a simulated escape exercise, for which data was gathered on time (sec) to complete the escape (from the article “Oxygen Consumption and Ventilation During Escape from an Offshore Platform,” Ergonomics, 1997: 281-292). The sample mean escape time for 26 observations is 370.69, and \( \sigma = 24.36 \).

   a) Calculate an upper confidence bound for population mean escape time using a confidence level of 95%. Express this confidence bound with a sentence.
   
   b) Calculate a 95% upper prediction bound for the escape time of a single worker.