1. Definitions you should know

1.1. Parameter
   1.1.1. A number that describes the population. (This is the definition, what follows are other statements that describe parameters.)
   1.1.2. Fixed (not random), but unknown. (This is an assumption underlying classical statistics, which is what we study in this class. In Bayesian statistics, the underlying assumption is that all unknown quantities are random variables.)

1.2. Statistic
   1.2.1. A number that describes the sample. (This is the definition, what follows are other statements that describe statistics.)
   1.2.2. A function of random variables, so it is also a random variable.
      1.2.2.1. Not limited to linear combinations of random variables, though linear combinations are easy to work with (or, in more formal language, linear combinations are quite tractable).
   1.2.3. Some of the most important statistics we’ll deal with in this class:
      1.2.3.1. The sample mean $\overline{X_n}$, which we usually use to estimate the population mean.
      1.2.3.2. The sample total
      1.2.3.3. The $Z$ statistic: $Z = \frac{\overline{X_n} - \mu}{\sigma_X / \sqrt{n}}$.
      1.2.3.4. The sample variance $s^2$ or standard deviation $s$
      1.2.3.5. The $T$ statistic (we didn’t talk about this one yet, but we will):
                           $T = \frac{\overline{X_n} - \mu}{s_X / \sqrt{n}}$
      1.2.3.6. If we’re interested in a population proportion, and we have the binomial setting, the sample proportion $\overline{X} / n$, where $X$ denotes the number of successes. We use the sample proportion to estimate the population proportion $\pi$, also known as the probability of success.
1.3. Random sample
1.3.1. The random variables $X_1, \ldots, X_n$ are considered to be a random sample from the population of interest if
1.3.1.1. They are independent.
1.3.1.2. They are identically distributed.

1.4. Sampling distribution
1.4.1. The probability distribution of a statistic.
1.4.2. What creatures have sampling distributions?

1.5. Linear combination of random variables (are examples of _____________?)
1.5.1. $\sum_{i=1}^{n} a_i X_i$

2. Random samples
2.1. Define a random sample
2.2. Recognize the notation $X_1, \ldots, X_n \sim \text{Dist}(\text{parm1}, \text{parm2})$ as defining a random sample.
2.3. Recognize when the problem situation has a random sample in it.
2.3.1. Because you were explicitly told it does.
2.3.2. Because, as in the binomial situation, you know that it does or that it does approximately based on characteristics of the sampling given in the problem.
2.4. Recognize when you need to assume that the problem situation involves a random sample, even though you were not given this information.
2.5. Whenever you assume the problem situation involves a random sample, provide justification for doing so.
2.5.1. You may have had to make this assumption to use the methods you learned in this course, but is it realistic to make that assumption?
2.5.2. If not, and if the problem is very important, there are methods that you could use that allow for violation of the identically distributed part, and/or for the violation of the independence part, and you may need to consult a statistical consultant, read some books, or take another stat class or two.
3. Sampling Distributions

3.1. Define sampling distribution (of a statistic)

3.2. Know what creatures have sampling distributions (see above)

3.3. Know and use the following methods for determining the sampling distribution of a statistic

3.3.1. For a discrete random variable with not very many possible values, the sampling distributions of some statistics can be derived by writing out every possibility—possibly in a clever way as in the following examples / problems we did

3.3.1.1. Example 5.20 from Sampling Distribution Notes Part 1 (sample means)
3.3.1.2. Sampling Distribution Assignment 2 (sample total and sample maximum)

3.3.2. Work out the theory and/or use theorems that I taught you (see below under the heading Theorems).

3.3.3. Use a simulation study

3.3.3.1. We might do this for the distribution of the sample proportion for the binomial distribution using the bead example.

4. Use your calculator to do the following:

4.1. Calculate normal probabilities (less than, greater than, in-betweens) given mean, variance or standard deviation, and the value of a random variable. (normcdf problems)

4.2. Given a mean, variance or standard deviation, and the normal probability that is bounded by one or two values of the normal random variable, be able to find the value(s) of the normal random variable. (invnorm problems)

5. Theorems about the sampling distributions of statistics based on certain populations—what you should be able to do with the theorems.

5.1. Be able to state the theorems—even though I allow a formula sheet, it is in your best interest to know these theorems and not be totally reliant on your formula sheet.

5.2. Recognize when one of the theorems below is and is not applicable to a problem situation.

5.3. If a theorem is not applicable to a problem situation, be able to tell what condition of the theorem is not met.

5.4. Determine which of the theorems below most completely matches the conditions given in the problem situation, and use that theorem to do the problem as quickly as possible, without having to do further derivations or proofs.
6. Theorems about the sampling distributions of statistics based on certain populations—statements of the theorems.

6.1. Mean and variance of a linear combination of random variables:

Let random variables \( X_1, \ldots, X_n \) have means \( \mu_1, \ldots, \mu_n \), and variances \( \sigma^2_1, \ldots, \sigma^2_n \), respectively. Then

6.1.1. Whether or not the \( X_i \) are independent,
\[ E(a_1X_1 + \cdots + a_nX_n) = a_1E(X_1) + \cdots + a_nE(X_n) = a_1\mu_1 + \cdots + a_n\mu_n \]

6.1.2. If the \( X_i \) are independent,
\[ \text{Var}(a_1X_1 + \cdots + a_nX_n) = a_1^2\text{Var}(X_1) + \cdots + a_n^2\text{Var}(X_n) = a_1^2\sigma^2_1 + \cdots + a_n^2\sigma^2_n \]

6.1.3. For any \( X_1, \ldots, X_n \),
\[ \text{Var}(a_1X_1 + \cdots + a_nX_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_ia_j \text{Cov}(X_i, X_j) \]

6.2. If \( X_1, \ldots, X_n \) are independent, normally distributed random variables—not necessarily identically distributed—then any linear combination of the \( X_i \) also has a normal distribution.

6.3. From the previous two theorems, we have that if \( X_1, \ldots, X_n \) are independently distributed normal random variables with means \( \mu_1, \ldots, \mu_n \) and variances \( \sigma^2_1, \ldots, \sigma^2_n \), respectively—or written another way, if, for \( i = 1, \ldots, n \), \( X_i \sim \text{N}(\mu_i, \sigma^2_i) \)—then
\[ \sum_{i=1}^{n} a_iX_i \sim \text{N} \left( \sum_{i=1}^{n} a_i\mu_i, \sum_{i=1}^{n} a_i^2\sigma^2_i \right) \]
Note that \( \sum_{i=1}^{n} a_iX_i \) is one new random variable.
Let \( X_1, \ldots, X_n \) be a random sample from any distribution with mean value \( \mu \) and variance \( \sigma_X^2 \). Written another way, for \( i = 1, \ldots, n \), \( X_i \sim \mathcal{N}(\mu, \sigma_X^2) \). Denote with \( T_0 \) the sample total \( T_0 = \sum_{i=1}^{n} X_i \). Then,

- **6.4.1.** \( E(\bar{X}_n) = \mu_{\bar{X}_n} = \mu \) --we actually only need that the RV's have the same mean for this to be true. We don't need them to have the same variance or be independent.

- **6.4.2.** \( V(\bar{X}_n) = \sigma_{\bar{X}_n}^2 = \sigma_X^2 / n \) --we need that the RV's have the same variance and are independent for this to be true. We don't need them to have the same mean.

- **6.4.3.** \( E(T_0) = \mu_{T_0} = n\mu \) --we actually only need that the RV's have the same mean for this to be true. We don't need them to have the same variance or be independent.

- **6.4.4.** \( V(T_0) = \sigma_{T_0}^2 = n\sigma_X^2 \) --we need that the RV's have the same variance and are independent for this to be true. We don't need them to have the same mean.

- **6.4.5.** \( E\left( \sum_{i=1}^{n} a_i X_i \right) = \mu \sum_{i=1}^{n} a_i \) --we actually only need that the RV's have the same mean for this to be true. We don't need them to have the same variance or be independent.

- **6.4.6.** \( V\left( \sum_{i=1}^{n} a_i X_i \right) = \sigma_X^2 \sum_{i=1}^{n} a_i^2 \) \( V(T_0) = \sigma_{T_0}^2 = n\sigma_X^2 \) --we need that the RV's have the same variance and are independent for this to be true. We don't need them to have the same mean.

**6.4.7.** When all of the conditions in 5.4 are met, then all of the above statements are true. When some of the conditions in 5.4 met, then some of the statements are true based on previous theorems.
6.5. Let $X_1, \ldots, X_n$ be a random sample from a **normal** distribution with mean value $\mu$ and variance $\sigma^2_X$. Written another way, for $i = 1, \ldots, n$, $X_i \sim \mathcal{N}(\mu, \sigma^2_X)$. Denote with $T_0$ the sample total $T_0 = \sum_{i=1}^{n} X_i$. Then

6.5.1. $\bar{X}_n \sim \mathcal{N}(\mu, \frac{\sigma^2_X}{n})$, and

6.5.2. $T_0 \sim \mathcal{N}(n \mu, n \sigma^2_X)$,

6.5.3. $\sum_{i=1}^{n} a_i X_i \sim \mathcal{N}(\mu \sum_{i=1}^{n} a_i, \sigma^2_X \sum_{i=1}^{n} a_i^2)$

6.6. Central Limit Theorem (CLT): Let $X_1, \ldots, X_n$ be a random sample from **any** distribution with mean value $\mu$ and variance $\sigma^2_X$. Written another way, for $i = 1, \ldots, n$, $X_i \sim \mathcal{N}(\mu, \sigma^2_X)$. Denote with $T_0$ the sample total $T_0 = \sum_{i=1}^{n} X_i$. Then, if $n$ is sufficiently large,

6.6.1. $\bar{X}_n \overset{\text{approx}}{\sim} \mathcal{N}(\mu, \frac{\sigma^2_X}{n})$,

6.6.2. $\sum_{i=1}^{n} a_i X_i \overset{\text{approx}}{\sim} \mathcal{N}(\mu \sum_{i=1}^{n} a_i, \sigma^2_X \sum_{i=1}^{n} a_i^2)$, provided that there are not one or two of the $a_i$ that are very large compared to the others. When in doubt, do a simulation study.