1 Hands-On Exercises#1

1.1 A – a different quadratic formula

One of more important concepts in algebra is the quadratic equation. To find roots of the quadratic equation of the form

\[ ax^2 + bx + c = 0 \]

you probably learned the quadratic formula:

\[ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

But there is another form of the quadratic formula that you probably didn’t learn in high school algebra:

\[ r = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}. \]

To check this out, find the root of the following quadratic equations using both formulas:

1. \( 6x^2 - 11x + 4 = 0 \)
2. \( 4x^2 - 13x + 3 = 0 \)
3. \( 2x^2 - x - 6 = 0 \)

1.2 B – mortgage payments

One of the rites of passage in the US is buying a house. For most Americans, the most mysterious thing when you buy a house is the calculation of the mortgage payment – the monthly payment for paying off a mortgage over a span of so many years, at a particular annual interest rate. (And I don’t want to get into variable rates and Jumbo mortgages!) Actually, the formula is simple:

\[ M = \frac{P \cdot r(1 + r)^N}{(1 + r)^N - 1} \]

where \( P \) is the Principal (amount you’re borrowing), \( r \) is the (monthly!) interest rate, and \( N \) is the number of months that you will be paying. Note that interest rates are usually given as an annual rate, so the practice is to divide by 12. Compute the mortgage payments for the following scenarios:

1. Borrowing $100,000 at 6% (annual) for 360 months (30 years)
2. Borrowing $250,000 at 4% (annual) for 120 months (10 years)
3. Borrowing $150,000 at 8.5% (annual) for 180 months (15 years)
1.3 C – How far can you see?

For someone whose eyelevel is at $h$ above ground/sea level, how far away is the horizon? Taking $r$ is the radius of the earth (3959 miles), the angle $\theta$ between the radius from the center of the earth to the observer and the radius to the tangent at the horizon satisfies $\cos(\theta) = r/(r + h)$. The distance – how far we can see – along the surface of the earth will be $r\theta$.

For various heights, compute the distance. Compare your solution to the approximation $\sqrt{2rh}$. In an airline magazine, I saw the formula $1.22\sqrt{\text{height in feet}}$ with the result in miles (1 mile=5280 feet); does this look right?

jfm, 21 August 2013