1 Hands-On Exercises #4 (30 August 2013)

1.1 A – symmetric square root

The Cholesky decomposition will give one type of square root of a positive definite matrix \( A \). Cholesky produces the upper triangular matrix \( R \), so that \( A = R^T R \). Another (different) square root can be constructed from the spectral (eigenvalue/vector) decomposition that is symmetric, usually denoted as \( A^{1/2} \). While there is another form of Cholesky, the symmetric square root is unique.

To make this symmetric square root for a positive definite matrix \( A \), we need the decomposition of \( A = UDU^T \), where \( D \) is the diagonal matrix of eigenvalues, and \( U \) is the (orthogonal) matrix whose columns are eigenvectors of \( A \). Then construct the symmetric square root as

\[
A^{1/2} = UD^{1/2}U^T
\]

where the square root of the diagonal matrix \( D \) is just the diagonal matrix of the square roots of the eigenvalues. If we multiply \( A^{1/2}A^{1/2} \), we will get the original matrix \( A \). Compute the symmetric square root for the following matrices:

1. \[
\begin{bmatrix}
4 & 2 & 2 & 4 \\
2 & 5 & 7 & 0 \\
2 & 7 & 19 & 11 \\
4 & 0 & 11 & 25
\end{bmatrix}
\]

2. \[
\begin{bmatrix}
10 & 3 & 2 \\
3 & 9 & 3 \\
2 & 3 & 19
\end{bmatrix}
\]

3. \[
\begin{bmatrix}
1 & -1 & 2 & 0 \\
-1 & 5 & -4 & 0 \\
2 & -4 & 6 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]

4. The matrix \( V \) so that \( (V)_{ij} = \exp(1 - |i - j|/8) \) for \( i, j = 1, \ldots, n \).

1.2 B – regression calculations

Fit a simple linear regression model \( E(y_i) = \beta_0 + \beta_1 x_i \), for \( i = 1, \ldots, n = 10 \) using the data below:

\[
x^T = [ 5.8 \ 5.8 \ 6.4 \ 6.4 \ 7.0 \ 7.0 \ 7.6 \ 7.6 \ 8.2 \ 8.2 ]
\]

\[
y^T = [ 68.8 \ 66.8 \ 66.4 \ 60.4 \ 68.0 \ 84.0 \ 73.6 \ 67.6 \ 83.2 \ 81.2 ]
\]

Compute the coefficient estimates, error sum of squares, residuals, and fitted values.

jf, 30 August 2013