1 Homework #3 – due 09 October 2013

In Homework #2, we looked at testing whether the rate of completion of repair tasks was the same for individual workers as for those working in pairs. I am concerned the distribution of the likelihood ratio statistic.

\[ T = -2(\ell_0(\hat{\theta}) - \ell(\hat{\lambda}_1, y_1) - \ell(\hat{\lambda}_2, y_2)) \]

1. First compute the p-value for the value of the statistic that you previously computed – let’s call it \( T_0 \). Under some standard assumptions, \( T \) ought to have a chi-square distribution with 1 df. Compute \( \Pr(X > T_0 | X \sim \chi^2_1) \).

2. Write the procedure that you did in Homework #2 as a single function that takes as its argument a single vector of length 12, with \( y_1 \) as its first 5 components, and \( y_2 \) as its last 7 components, and produces just the likelihood ratio statistic \( T \) as its result. Test it with the data from Homework #2.

3. Generate a matrix with 5 rows and \( N \) columns where each element is iid Poisson with rate \( \lambda_1 \). Then generate a second matrix with 7 rows and \( N \) columns with Poisson random variables with rate \( \lambda_2 \). Then put them together with \( \text{rbind} \) and call the matrix \( Y \).

4. Compute the likelihood ratio statistic \( T_j \) for each column \( j \) of the \( Y \).

5. Count how many of these test statistics \( T_j \) are greater than \( T_0 \).

This procedure is called the parametric bootstrap. What to use for \((\lambda_1, \lambda_2)\)? Well, what we want is the distribution of the test statistic when the hypothesis is true, so we need \( \lambda_2 = \frac{1}{2} \lambda_1 \). And we want this close to the problem that we have, so one choice is \((\hat{\lambda}_1, 2\hat{\lambda}_1)\), a second is \((\hat{\lambda}_2/2, \hat{\lambda}_2)\), and a third is \((\hat{\theta}, 2\hat{\theta})\).