1 Hands-On Exercises #12 (04 and 09 October 2013)

The figures for my solutions are posted in a directory linked nearby to this.

1.1 A – bounds on the t-density

Graph the three functions below to show visually the following inequalities regarding the unnormalized density of the Student’s t distribution, \( u_\alpha(x) = (1 + x^2/\alpha)^{-(\alpha+1)/2} \).

\[
\begin{align*}
u_\alpha(x) &\leq [2u_\alpha(1)] u_1(x) \leq [2u_\infty(x)] u_1(x) = 2e^{-1/2}/(1 + x^2) 
\end{align*}
\]

Just graph the three functions for one value of \( \alpha \). If you get more ambitious, make a multiple panel plot for many values of \( \alpha \).

1.2 B – sleeping dog data

In H-O#9B, you looked at the sleeping dog data. With the response as Y and treatment combination (1 to 4) as X, connect the dots for each of the 19 dogs.

Here the legend is critical, as the viewer will need to distinguish among the many dogs.

Some annoying details:

- there are only 6 different line types (lty) (not including null)
- if you want different colors, then you may need to compute all of the colors (with rainbow(howmany) or heat.colors(howmany)) and store them in a vector for later access

1.3 C – salary data

In H-O#9C, you looked at salary data for males and females (M,F) for different levels of experience. Plot the data, distinguishing gender, and overlay parallel lines for the two groups.

1.4 D – Poisson regression

In H-O#10B and #11B, you looked at the Poisson regression model with the log link to the mean function. The log-likelihood for this model takes the form

\[
\ell(\beta_0, \beta_1) = \sum_{i=1}^{n} [y_i \log(\lambda_i) - \lambda_i - \log(y_i!)] ,
\]

where \( \lambda_i = \exp(\beta_0 + \beta_1 x_i) \). Draw a contour plot or an image plot of the log-likelihood function (a function of \( \beta_0 \) and \( \beta_1 \)) for the case where the design points
are \( x = (1, 2, 3, 4, 5, 6) \) and the responses are \( y = (1, 0, 1, 4, 4, 3) \). For what values of \((\beta_0, \beta_1)\)? Find the MLE with

\[
glm(y, x, \text{family} = \text{poisson(link = "log")})
\]

and go out two standard errors for the two parameters.

### 1.5 E – from Homeworks #2 & #3

Draw the three likelihood functions from Homeworks #2 and #3: \( \ell(\theta, y_1) \), \( \ell(2\theta, y_2) \), and \( \ell_*(\theta) \). Note that all three use the same parameter \( \theta \), corresponding to \( \lambda_1 \) with the \( y_1 \) data, \( \lambda_2/2 \) with the \( y_2 \) data.

jfm, 08 October 2013