1 Hands-On Exercises #7 (13 September 2013)

1.1 A – finding quantiles

Underlying most of R’s quantile functions is really a root-finding problem. The root of the function $P(x) - \alpha$ finds the $\alpha$ quantile of a distribution.

1. For the normal distribution, find the root of $P(x) - \alpha$ for $\alpha = .01, .025, .05, .10, .25$ and compare your results to the R function `qnorm(\alpha)`.

2. In a similar fashion, find the $\alpha = .05$ critical value for Student’s t distribution for values of the degrees of freedom parameter, $k = 1, 2, 3, 5, 10$ and compare to `qt(.05, k)`.

3. For the same list of $\alpha$’s and $t = 1, 2, 3$, compute the percentile points (quantiles) of the inverse Gaussian distribution with distribution function

$$
\Phi((x-1)\sqrt{t/x}) + e^{2t}\Phi(-(x+1)\sqrt{t/x})
$$

for $x > 0$ where $\Phi(x)$ is the distribution function of the normal distribution.

1.2 B – simple L1 regression

For the simple linear regression problem $y_i = b_0 + b_1x_i + e_i$ with iid logistic errors $e_i$ (rather than normal), then the MLE for the regression coefficients minimize the L1 error function

$$
S(b_0, b_1) = \sum_{i=1}^{n} |y_i - b_0 - b_1x_i|.
$$

Instead of writing a function of two parameters to be minimized, we know that we can find the minimum value of $b_0$ for any value of $b_1$ as the median of $f_i = y_i - b_1x_i$. Write a function (of $b_1$ only) that first computes $f_i$, then its median $m$, and then has $\sum |f_i - m|$ as the result of the function. Using $y_i$ and $x_i$ from H-O #4b and compute L1 coefficient estimators and compare to the usual (L2) least squares estimators.

jfm, 13 September 2013