1 Hands-On Exercises #8 (18 September 2013)

1.1 A – matrix norms

Matrix norms are used to measure the size of a matrix and two of them are particularly useful in numerical linear algebra.

- The column sum norm for the matrix $A$ is $\|A\|_1 = \max_j \sum_{i=1}^n |A_{ij}|$ and is used with the $p=1$ vector norm.
- The row sum norm for the matrix $A$ is $\|A\|_\infty = \max_i \sum_{j=1}^n |A_{ij}|$ and is used with the $p=\infty$ vector norm.

Write functions to compute the column and row sum matrix norms.

1.2 B – more matrix norms

These norms are supposed to be largest value of the ratio $\|Ax\|_p/\|x\|_p$ where $\|x\|_p = (\sum |x_i|^p)^{1/p}$ which obviously simplifies to a sum of absolute values for $p = 1$, and simplifies to $\max_i |x_i|$ for $p = \infty$. For one of these two norms, find the largest value as you search over $N$ randomly distributed vectors $x$.

1. Compute the norm $\|A\|_p$ of the matrix $A$ for either $p = 1$ or $p = \infty$.
2. Write a function to compute the ratio $\|Ax\|_p/\|x\|_p$ for a matrix $A$ with the vector $x$ as the argument.
3. For $N$ large (like $10^3$ or more), generate a matrix of normals, so that each column is a random vector from a multivariate standard normal (each component iid $N(0,1)$) (and conformable with $A$).
4. Compute the ratios for each column of this matrix.
5. Find the largest values of these ratios. For what vector (think which.max) do we get the largest ratio?

1.3 B – Rayleigh quotients

For a matrix $A$, a Rayleigh quotient in a vector $x$ is the quadratic form in that matrix divided by its squared length: $\frac{x^T A x}{x^T x}$. For a symmetric matrix, the extreme points of the Rayleigh quotient are the eigenvalues; what happens if the matrix is not symmetric?

As before, compute the eigenvalues of the matrix $A$ and also its symmetrized version $(A + A^T)/2$. Write a function to compute the Rayleigh quotient for a matrix $A$ with the vector $x$ as the argument. Generate a matrix of normals, and compute the Rayleigh quotients for each column of this matrix. Find the largest and smallest values of these quotients. Are they close to the eigenvalues of the original matrix $A$ or its symmetrized version?
Use the asymmetric matrix $A = \begin{bmatrix} 5 & 4 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 4 \end{bmatrix}$