ST 552 Linear Models Spring 2008

Homework #2 -- due Friday, 25 January 2008 *** turn in just starred (*) questions ***

Exercises at the end of Appendix A: 12, 16, 22, 23, 24, 36, 46, 53*, 54*

*1) (Turn in this one!) Let $\mathbf{A} = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 2 & 0 \\ 4 & 0 & 4 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

a) Show that $\mathbf{c}$ is in $\mathcal{C}(\mathbf{A})$. (Find a vector $\mathbf{x}$ such that $\mathbf{A}\,\mathbf{x} = \mathbf{c}$.)

b) Find two different generalized inverses, say $\mathbf{G}_1$ and $\mathbf{G}_2$ for $\mathbf{A}$.

c) For one of your gi's (say, $\mathbf{G}_1$)in (b), compute $\mathbf{A}\mathbf{G}_1$.

d) Is $\mathbf{A}\mathbf{G}_1$ from (c) idempotent? symmetric?

e) Let $\mathbf{x}^* = \mathbf{G}_1\mathbf{c}$; show that $\mathbf{A}\mathbf{x}^* = \mathbf{c}$.

f) Show that $\mathbf{G}_2\mathbf{c}$ also solves $\mathbf{A}\,\mathbf{x} = \mathbf{c}$.

g) Find $\mathbf{z}$ so that $\mathbf{G}_2\mathbf{c} = \mathbf{G}_1\mathbf{c} + (\mathbf{I} - \mathbf{G}_1\mathbf{A})\mathbf{z}$.

2) Find the eigenvalues and describe the eigenvectors for the following matrix:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3) Suppose the matrices $\mathbf{A}$ and $\mathbf{B}$ are square, $n \times n$, and $\mathbf{A}\mathbf{B} = \mathbf{I}_n$. Can you prove that $\mathbf{A}$ is nonsingular and $\mathbf{B} = \mathbf{A}^{-1}$?