

Homework #5 -- due Friday, 29 February 2008 \*\*\* turn in just starred (\*) questions \*\*\*

Exercises at the end of Appendix A: 70, 71\*, 72

Exercises at the end of Chapter 3: 23, 24, 26

Exercises at the end of Chapter 4: 1, 2, 6\*, 10, 14, 25

1) Show that if  $\mathbf{A}$  is nonsingular, then

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1} & \mathbf{A}^{-1}\mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1} \\ (\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1} & -(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1} \end{bmatrix}$$

\*2) Consider the following unbalanced one-way analysis of variance model

$$y_{ij} = \mu + \alpha_i + e_{ij}, \quad i = 1, 2, 3 \text{ and } j = 1, \dots, n_i = (2i),$$

so that  $N = n_1 + n_2 + n_3 = 2 + 4 + 6 = 12$ .

j	1	2	3	4	5	6	$y_i$
$y_{1j}$	6	8					14
$y_{2j}$	10	9	9	12			40
$y_{3j}$	7	6	6	8	9	6	42

a) Find all solutions to the normal equations.

b) Consider the constraint  $2\alpha_1 + 4\alpha_2 + 6\alpha_3 = 0$ . Write it as  $\mathbf{P}^T\mathbf{b} = \boldsymbol{\delta}$ . by giving  $\mathbf{P}$  and  $\boldsymbol{\delta}$ . Is  $\mathbf{P}^T\mathbf{b}$  estimable?

c) Find a solution to the restricted normal equations employing this constraint. Is the Lagrange multiplier zero?

d) Consider the constraint  $\alpha_1 - \alpha_2 = 0$  and  $\alpha_2 - \alpha_3 = 0$ , and, again, write it as  $\mathbf{P}^T\mathbf{b} = \boldsymbol{\delta}$ . by giving  $\mathbf{P}$  and  $\boldsymbol{\delta}$ . Are the components of  $\mathbf{P}^T\mathbf{b}$  estimable?

e) Again, find a solution to the restricted normal equations employing this constraint. Is the Lagrange multiplier zero?