

Homework #2 -- due Friday, 23 January 2009 \*\*\* turn in just starred (\*) questions \*\*\*

Exercises at the end of Appendix A: 12, 16, 23, 24, 25\*, 36, 46, 53\*, 54

\*1) (Turn in this one!) Let  $\mathbf{A} = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$  and  $\mathbf{c} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

- Show that  $\mathbf{c}$  is in  $\mathcal{C}(\mathbf{A})$ . (Find a vector  $\mathbf{x}$  such that  $\mathbf{A}\mathbf{x} = \mathbf{c}$ .)
- Find two different generalized inverses, say  $\mathbf{G}_1$  and  $\mathbf{G}_2$  for  $\mathbf{A}$ .
- For one of your generalized inverses in (b), say,  $\mathbf{G}_1$ , compute  $\mathbf{A}\mathbf{G}_1$ .
- Is  $\mathbf{A}\mathbf{G}_1$  from (c) idempotent? symmetric?
- Let  $\mathbf{x}^* = \mathbf{G}_1\mathbf{c}$ ; show that  $\mathbf{A}\mathbf{x}^* = \mathbf{c}$ .
- Show that  $\mathbf{G}_2\mathbf{c}$  also solves  $\mathbf{A}\mathbf{x} = \mathbf{c}$ .
- Find  $\mathbf{z}$  so that  $\mathbf{G}_2\mathbf{c} = \mathbf{G}_1\mathbf{c} + (\mathbf{I} - \mathbf{G}_1\mathbf{A})\mathbf{z}$ .

2) Find the eigenvalues and describe the eigenvectors for the following matrix:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) Suppose the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are square,  $n \times n$ , and  $\mathbf{A}\mathbf{B} = \mathbf{I}_n$ . Can you prove that  $\mathbf{A}$  is nonsingular and  $\mathbf{B} = \mathbf{A}^{-1}$ ?