ST 552 Linear Models Spring 2009

Homework #5 * REVISED * -- due Friday, 27 February 2009 *** turn in just starred (*) questions ***

Exercises at the end of Appendix A: 70, 71*, 72

Exercises at the end of Chapter 3: 23, 24, 26

Exercises at the end of Chapter 4: 1, 2, 6*, 10, 14, 25

1) Show that if **A** is nonsingular, then

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1} & \mathbf{A}^{-1}\mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1} \\ (\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1} & -(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1} \end{bmatrix}$$

*2) Consider the following unbalanced one-way analysis of variance model

$y_{ij} = \mu + \alpha_i + e_{ij}$, $i = 1, 2, 3$ and $j = 1, ..., n_i$, where $n_1 = 2$, $n_2 = 3$, $n_3 = 4$,

so that $N = n_1 + n_2 + n_3 = 2 + 3 + 4 = 9$.

j	1	2	3	4	$y_{i.}$
y_{1j}	6	8			14
y_{2j}	10	9	11		30
y_{3j}	7	6	6	9	28

a) Find all solutions to the normal equations.

b) Consider the constraint $2\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0$. Write it as $\mathbf{P}^T\mathbf{b} = \boldsymbol{\delta}$. by giving **P** and $\boldsymbol{\delta}$. Is $\mathbf{P}^T\mathbf{b}$ estimable?

c) Find a solution to the restricted normal equations employing this constraint. Is the Lagrange multiplier zero?

d) Show that all components of **b** are estimable by showing that the (10×4) matrix $(\mathbf{X}^T \;\; \mathbf{P})$ has full row rank.

e) Show that $\bar{y}_{1.} - \bar{y}_{..}$ is an unbiased estimator for (estimable!) α_1 in the restricted parameter space $\mathcal{T}_1 = \{ (\mu, \alpha_1, \alpha_2, \alpha_3) \mid 2\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0 \}$.

f) Consider the new contraint $\alpha_1 - \alpha_2 = 0$ and $\alpha_2 - \alpha_3 = 0$, and, again, write it as $\mathbf{P}^T\mathbf{b} = \boldsymbol{\delta}$. by giving **P** and $\boldsymbol{\delta}$. Are the components of $\mathbf{P}^T\mathbf{b}$ estimable?

e) Again, find a solution to the restricted normal equations employing this new constraint. Is the Lagrange multiplier zero?