1) Show that if $A$ is nonsingular, then
\[
\begin{bmatrix}
    A & B \\
    B^T & 0
\end{bmatrix}^{-1} = \begin{bmatrix}
    A^{-1} - A^{-1}B(B^T A^{-1}B)^{-1}B^TA^{-1} & A^{-1}B(\beta^T A^{-1}B)^{-1} \\
    (B^T A^{-1}B)^{-1}B^T A^{-1} & -(B^T A^{-1}B)^{-1}
\end{bmatrix}
\]

*2) Consider the following unbalanced one-way analysis of variance model
\[
y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad i = 1, 2, 3 \text{ and } j = 1, ..., n_i, \text{ where } n_1 = 2, n_2 = 3, n_3 = 4,
\]
so that $N = n_1 + n_2 + n_3 = 2 + 3 + 4 = 9.$

<table>
<thead>
<tr>
<th>j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>y_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>y_{1j}</td>
<td>6</td>
<td>8</td>
<td></td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>y_{2j}</td>
<td>10</td>
<td>9</td>
<td>11</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>y_{3j}</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>28</td>
</tr>
</tbody>
</table>

a) Find all solutions to the normal equations.

b) Consider the constraint $2\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0$. Write it as $P^T \beta = \delta$. Is $P^T \beta$ estimable?

c) Find a solution to the restricted normal equations employing this constraint. Is the Lagrange multiplier zero?

d) Show that all components of $\beta$ are estimable by showing that the $(10 \times 4)$ matrix $(X^T P)$ has full row rank.

e) Show that $\overline{y}_1 - \overline{y}_. \alpha_1$ is an unbiased estimator for (estimable!) $\alpha_1$ in the restricted parameter space $T_1 = \{ (\mu, \alpha_1, \alpha_2, \alpha_3) \mid 2\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0 \}$.

f) Consider the new constraint $\alpha_1 - \alpha_2 = 0 \text{ and } \alpha_2 - \alpha_3 = 0$, and, again, write it as $P^T \beta = \delta$. Are the components of $P^T \beta$ estimable?

e) Again, find a solution to the restricted normal equations employing this new constraint. Is the Lagrange multiplier zero?