1. (Exercise taken from p. 685 of Rao’s “Statistical Research Methods in the Life Sciences”):

- See data file called “wether.dat”
- Study investigates three diet supplements on weights of wethers (rams)
- Four randomly selected locations (i.e., environments)
- 24 wethers randomly assigned to locations in a balanced design
- In each location, all three diets used, two animals per diet.

(a) Propose a model for these data: \( Y_{ijk} = \) ?
   - Fixed, random or mixed?
   - Nested or crossed?

(b) Obtain two scatterplots with mean weight gain on the vertical axis and:
   - location on the horizontal axis with different symbols for different diets
   - diet on the horizontal axis with different symbols for different locations

   ```
symbol1 i=join value=dot;
proc means nway noprint; /* nway suppresses marginal means */
class diet location;
    vary y;
    output out=two mean=ymean;
run;

proc gplot;
    plot ymean*location=diet;
    plot ymean*diet=location;
run;
```

(c) Inspect the plots
   - Does the first plot provide evidence of a location effect?
   - a location × diet effect?
   - Does the second plot provide evidence of a diet effect?
   - a location × diet effect?

(d) Investigate these effects by fitting a two-factor model with interaction using `PROC GLM`. Note that if the location effect is random, the \( p \)-values from \( F \)-tests may not be right.
(e) Further investigate these effects by fitting a mixed model using the code below:

```plaintext
proc mixed method=type3;
  class location diet;
  model wtgain=diet/ddfm=satterth;
  random location location*diet;
run;
```

i. Find the correct $F$-ratio and $p$-values to test for
   A. A random diet $\times$ location interaction effect
   B. A random location effect
   C. A fixed diet effect

ii. Find the estimated variance components which quantify the magnitude of the first two of these effects above (d)iA (d)iB.

iii. For each of the three variance components, obtain Wald 95% confidence intervals of the form

\[
\left( \frac{\hat{df}\hat{\sigma}_v^2}{\chi^2(\alpha = 0.025)} \right)^{1/2} \left( \frac{\hat{df}\hat{\sigma}_v^2}{\chi^2(\alpha = 0.975)} \right)^{1/2}
\]

using the cl option in the `proc mixed` statement:

```plaintext
proc mixed cl; /* without method=type3 option */
```

iv. As in problem 14.16d, obtain simultaneous 90% confidence intervals for all 3 pairwise differences of diet means. (Satterthwaite formula for $\hat{df}$ associated w/ SE terms is unnecessary here.)

```plaintext
lsmeans diet/adj=bon cl alpha=0.1;
```

Conclusion: though there is evidence of a diet $\times$ location random interaction effect, so that the random location effects vary by diet, the diet main effect is not significant and none of the pairwise comparisons among diet means are significant, thus not providing much evidence of a fixed diet effect. Averaged over locations, mean weight gain is plausibly equal for the diets.