1. A doctor notes that 20% of the patients he tests actually have mononucleosis. When a patient has mono, the test shows a positive result (indicating disease presence) 90% of the time. When a patient does not have the disease, the test shows a positive result 10% of the time. If a patient's test result is positive, what is the probability that the patient actually has the disease?
   a. .18  b. .26  c. .69  d. .77  e. .90

2. Four radar systems are arranged so that they work independently of each other. Each system has a 0.9 chance of detecting an approaching airborne object. Find the probability that at least one radar system will fail to detect an approaching object.
   a. (.9)^4  b. (.1)^4  c. 1 - (.9)^4  d. 1 - (.1)^4  e. .9 + .9 - (.9)(.9) = .99

3. A federal agency is trying to decide which of two waste dump projects to investigate. An administrator estimates that the probability of federal law violations in the first project is 0.3. She also estimates that the probability of violations in the second project is 0.25. In addition, she believes the occurrence of violations in these two projects are mutually exclusive. The probability of federal law violations in the first project or in the second project or both is
   a. .075  b. .05  c. .55  d. none of these  e. can't tell from information given

4. A manufacturer of hand soap has introduced a new product. An extensive survey indicates that 40% of the people have seen advertising for the new product. It also showed that 20% of the people in the survey had tried the new product. In addition, 15% of those in the survey had seen it advertised and had tried the product. What is the probability that a randomly chosen person would have seen the advertising for the new product or have tried the product or both?
   a. .6  b. .45  c. .08  d. .25  e. .05

5. How many four-digit serial numbers can be formed if no digit is to be repeated within any number? (The first digit may be a zero).
   a. (10)^4  b. 10!  c. 10P_4  d. 5040  e. (10)^4

6. These questions are based on the Peanuts cartoon shown at the end of the questions.
   I. In how many ways can seven books be arranged on a bookshelf? (The order in which books are arranged matters).
      a. 7C_1  b. 7!  c. 7P_1  d. 7^7  e. 77
   II. If you have only 3 books to put on a bookshelf, in how many ways can three books be arranged?
       a. 3  b. 9  c. 6  d. 27  e. 33
   III. Circle your answer to the question posed in the third frame of the cartoon.
        a. 5040  b. 4! x 3!  c. 7(3!)  d. 720  e. 7(7!)

7. Mathcounts is a national mathematics competition for seventh- and eighth-graders. In 1992 the contest was won by Andrei Gnepp of Orange, Ohio by answering the following question about the National Basketball Association (NBA) playoffs:
   The Chicago Bulls lead the Detroit Pistons 3 games to 2 in a seven game playoff. Assuming the probability that the Bulls win any particular game against the Pistons is \( \frac{3}{5} \), what is the probability that the Pistons will win the playoff?
      a. \( \frac{2}{5} \)  b. \( \frac{2}{5} \)  c. \( 1 - \frac{9}{25} \)  d. \( \frac{4}{25} \)  e. Can't tell; depends on Michael Jordan's jersey #
8. Determine which of the following functions is in fact a probability distribution function.
   
   a. \( p(x) = \frac{1}{4}, x = 3, 4, 5, 6. \)
   
   b. \( p(x) = \frac{x^2}{25}, x = 0, 1, 2, 3, 4. \)
   
   c. \( p(x) = \frac{5-x^2}{6}, x = 0, 1, 2, 3. \)

9. In a population of students the number of calculators owned is a random variable \( x \) with \( p(0) = .2, p(1) = .6, p(2) = .2. \) Find the expected value and standard deviation of this probability distribution.

10. An oil firm is to drill three wells, with each well having probability 0.2 of successfully producing oil. It costs the firm $20,000 to drill each well. A successful well will bring in oil worth $750,000. Let the random variable \( X_i \) be the oil firm's gain from well \( i, i = 1, 2, 3. \) The wells are in different geographic areas and so the drilling outcome at any well has no affect on the drilling outcomes at the other wells.
   
   a. Find the firm's expected gain \( G \) from the three wells.
   
   b. Find the standard deviation of the firm's gain.

11. Let the random variable \( X \) denote the displacement in cubic inches \((in^3)\) of the engine in a particular model of automobile. The size of the engine (that is, the cubic inch displacement) varies depending on the options chosen by the buyer of the automobile. It is known that \( E(X) = 177 \) in\(^3\) and \( \sigma_X = 22 \) in\(^3\). If \( X^* \) denotes the engine displacement in cubic centimeters \((cm^3)\), determine \( E(X^*) \) and \( \sigma_{X^*} \). Note that 1 in\(^3\) = 16.4 cm\(^3\).

12. Let \( X \) be the number of accidents per week at a hazardous intersection; \( X \) varies with mean 2.2 and standard deviation 1.4. Let \( X_1, X_2, \) and \( X_3 \) be the number of accidents in each of 3 different weeks at this intersection. The number of accidents in a week is not affected by the number of accidents in any other week. What is \( \sigma_{(X_1+X_2+X_3)} \), the standard deviation of the sum \( X_1 + X_2 + X_3 \)?

13. The probability distribution below describes the number of repair calls that an appliance repair shop may receive during an hour.
   
<table>
<thead>
<tr>
<th>Repair calls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

   a. How many calls should the shop expect to receive per hour? What is the standard deviation?
   
   b. Find the expected value and standard deviation of the number of repair calls the appliance shop should expect during an 8-hour day.

14. A college student on a meal plan reports that the amount of money he spends daily on food varies with a mean (expected value) of $13.50 and a standard deviation of $7.
   
   a. Find the expected value and standard deviation of the amount he spends on 2 consecutive days (the amounts he spends on different days are independent).
   
   b. Find the expected value and standard deviation of the amount he spends during a semester that spans 120 days.

15. Which of the following best describes the origin of \( p^x(1-p)^{n-x} \) in the binomial probability function?
   
   a) It is the probability of each path with exactly \( x \) successes and \( n-x \) failures.
   
   b) It is the probability that the first success occurs in the \( x \)th trial among the \( n \) trials.
   
   c) It is the probability of \( x \) or more successes among the \( n \) trials.
   
   d) It is the probability of \( x \) or less successes among the \( n \) trials.

16. Which of the following does NOT characterize the setting of a binomial random variable?
   
   a) there are \( n \) identical trials
   
   b) the trials are dependent
   
   c) each trial has two possible outcomes
   
   d) the chance of "success" on each trial is the same
17. Which of the following best describes the origin of \( \binom{n}{x} \) in the binomial probability function?
   a) \( n! / x! \)
   b) It is the number of successes among the \( n \) trials.
   c) It is the number of paths with \( x \) successes and \( (n-x) \) failures.
   d) It is the number of ways that the first success can occur on the \( x \)th trial.

18. Which of the following statements is false?
   a) In a binomial experiment the \( n \) trials are independent.
   b) The standard deviation of a binomial random variable is \( np(1-p) \).
   c) The mean of a binomial random variable is \( np \).
   d) There are only two possible outcomes.

19. A telephone survey of American families is done to determine the number of children in the average American family. Past experience has shown that 30% of those telephoned will refuse to respond to the survey. Given the above information, which of the following is NOT a binomial random variable?
   a) The number of families responding in 50 calls.
   b) The number of families not responding in 50 calls.
   c) The number of children in a family that responds.
   d) All of the above are binomial random variables

20. Let \( x \) be a binomial random variable with \( n=20 \) and \( p=0.4 \). Which of the following is(are) correct?
   a) the mean of \( x \) is 12.
   b) the standard deviation of \( x \) is 4.8.
   c) \( \sum p(x) = 1 \), \( x=0,...,20 \)
   d) all of the above are correct
   e) none of the above are correct

21. To determine the effectiveness of a certain diet to reduce the amount of cholesterol in the blood stream, 20 people were put on a new diet. After they had been on the new diet for a sufficient length of time, their cholesterol count was taken. The person running the experiment will endorse this diet only if at least 16 out of 20 people lower their cholesterol count. What is the random variable of interest?
   a) \( X=16 \) people
   b) \( X=20 \) people
   c) \( X=\)at least 16 out of 20 people lowered their cholesterol counts after going on the diet.
   d) \( X=\# \) out of 20 people whose cholesterol count was lowered after going on the diet
   e) \( X=\)person whose cholesterol count was lowered after going on the diet.

22. To determine the effectiveness of a certain diet to reduce the amount of cholesterol in the blood stream, 20 people were put on a new diet. After they had been on the new diet for a sufficient length of time, their cholesterol count was taken. The person running the experiment will endorse this diet only if at least 16 out of 20 people lower their cholesterol count. What is the probability that the experimenter endorses the diet if, in fact, it has no effect on the cholesterol level (i.e., there is an equal chance that a person's cholesterol level will go up or down)?
   a) 0.999
   b) 0.006
   c) 0.001
   d) 0.005
   e) 0

23. A pet store owner sells specialty clothes for pets. From past data 5% of customers buy specialty clothes for their pets. What is the probability that at least 4 of the first 20 customers buy specialty clothes for their pets?
   a) 0.997    b) 0.984    c) 0.013    d) 0.016    e) 0.003
24. In a binomial experiment with $n$ trials let $p =$ probability of success, $q =$ probability of failure and $X =$ # of successes. Then:
   a) $p + q = 1$
   b) $\sum p(x) = 1, x = 0, ..., n$
   c) $P(X = 0) = q^n$
   d) $X \leq n$
   e) all of the above

25. A fair coin is tossed $n$ times. If you EXPECT to get 7 heads in the $n$ trials then you should toss the coin:
   a) 7 times
   b) $(0.5)^7$ times
   c) 14 times
   d) cannot be determine from the given information.

26. Identify the binomial experiment in the following group of statements.
   a) a shopping mall is interested in the income levels of its customers and is taking a survey to gather information
   b) a business firm introducing a new product wants to know how many purchases its clients will make each year
   c) a sociologist is researching an area in an effort to determine the proportion of households with male "head of household"
   d) a study is concerned with the average hours worked by teenagers who are attending high school

27. When $n$ is relatively small and $p$ close to 1, the binomial probability distribution is
   a) mound shaped
   b) skewed to the right
   c) skewed to the left
   d) rectangular

28. The owner of a small convenience store notices that only 5% of customers buy magazines.
   a. What is the probability that the first customer to buy a magazine is the 4th customer?
   b. What is the probability that the first customer to buy a magazine is the 8th customer?
   c. How many customers should the owner expect until a customer buys a magazine?

29. The colors of M&M candies in a typical bag have the following probabilities: $P(\text{brown}) = 0.30$, $P(\text{red}) = 0.20$, $P(\text{yellow}) = 0.20$, $P(\text{green}) = 0.10$, $P(\text{orange}) = 0.10$, $P(\text{blue}) = 0.10$. You have just purchased a bag of M&M's and select one candy at a time from the bag. What is the probability that the first red one is the 4th candy you select from the bag?

30. What is the expected number of tosses of a fair die until each of the six numbers appears at least once?

31. During the lunch-hour rush at a McDougalds restaurant orders for the Big Mac hamburger follow a Poisson distribution and occur at a rate of 4 per minute. What is the probability that 5 or more Big Macs will be ordered in one minute?

32. Which of the following is a correct statement concerning the Central Limit Theorem (CLT)?
   a) The CLT states that the sample mean, $\bar{x}$, is always equal to $\mu$.
   b) The CLT states that for large samples, the sampling distribution of the sample mean is approximately normal.
   c) The CLT states that for large samples, sample mean $\bar{x}$ is equal to $\mu$.
   d) The CLT states that for large samples, the sampling distribution of the population mean is approximately normal.
   e) Both c and d are correct.
33. The amount of money spent on food per week by a typical American family is known to have a mean of 92 dollars with a standard deviation $\sigma$ of 9 dollars. Suppose a random sample of 81 families is taken and the sample mean is calculated.
   a. Describe the sampling distribution model of the sample mean. (Include the mean, standard deviation, and type of distribution if known).
   b. Find the probability that the sample mean does not exceed 90.4 dollars.

34. In a learning experiment, untrained mice are placed in a maze and the time required for each mouse to exit the maze is recorded. For untrained mice, the average time to exit the maze is $\mu = 50$ sec and the standard deviation is $\sigma = 16$ sec. If 64 randomly selected untrained mice are placed in the maze and the time necessary to exit the maze recorded for each one, what is the probability that the sample mean differs from 50 by more than 3?

35. It is generally believed that nearsightedness affects about 12% of children. A school district gives vision tests to 133 incoming kindergarten children. What is the probability that over 15% of the children will be found to be nearsighted?

36. In the 1992 U.S. presidential election, Bill Clinton received 43% of the vote compared to 38% for George H. W. Bush and 19% for Ross Perot. Suppose we had taken a random sample of 100 voters in an exit poll and asked them for whom they had voted. In 95% of such polls, our sample proportion of voters for Clinton should be between what two values that are equidistant from the expected value?
Solutions

1. c.

\[
P(\text{patient actually has the disease if test result positive}) = \frac{18}{18+0.8} = \frac{18}{18.8} \approx 0.9
\]

2. c. since the probability that no radar system fails to detect an airborne object (i.e. all four radar systems work) is \(0.9^4\), therefore the probability that at least one fails is \(1 - 0.9^4\).

3. c

4. b. since \(P(\text{seen advertising tried the product}) = 0.4 + 0.2 = 0.6\) = 0.45.

5. c. and d. are both correct since \(10(9)(8)(7) = 5040 = 10P_4\).

6. I. b. \(7! = 5040\) II. c. \(3! = 6\)

III. d. View the problem as filling 5 slots: one slot for each of the four science book and one slot for the group of 3 math books. There are \(5!\) ways to fill these 5 slots since order makes a difference; in addition, the 3 math books can be arranged in \(3!\) ways. So there are \(3! \times 5! = 720\) ways.

7. The winner of the playoff is the first team to win 4 games. Therefore, the Pistons must win 2 games in a row. \(P(\text{Pistons win any particular game against Bulls}) = \frac{2}{3}\), so \(P(\text{Pistons win playoff}) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}\).

8. a. is the only one since in \(\sum_{i=1}^{\infty} p(x) > 1\); in c. \(p(3) < 0\).

9. \(\mu = 0(2.2) + 1(6.6) + 2(2.2) = 1\)

\(
\sigma = sqrt[(0 - 1)^2(0.2) + (1 - 1)^2(0.6) + (2 - 1)^2(0.2)] = sqrt[0.4] = 0.6325
\)

10. a. \(E(X_1) = 0.2 * 730,000 + 0.8 * (-20,000) = 130,000; G = X_1 + X_2 + X_3; E(G) = E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3) = 390,000.\)

b. \(Var(X_1) = \left[130,000 - 130,000\right]^2 * 0.2 + \left(-20,000 - 130,000\right)^2 * 0.8 = 9 \times 10^{10};\)

\(Var(G) = Var(X_1 + X_2 + X_3) = Var(X_1) + Var(X_2) + Var(X_3) = 27 \times 10^{10};\)

\(SD(G) = \sqrt{Var(G)} = \sqrt{27 \times 10^{10}} = 519,615.24.\)

11. \(E(X^2) = 177 \text{ in}^3 \times 16.4 \text{ cm in}^2 = 2902.8 \text{ cm}^3;\)

\(\sigma_X = 22 \text{ in}^3 \times 16.4 \text{ cm in}^2 = 360.8 \text{ cm}^3.\)

12. \(Var(X_1 + X_2 + X_3) = Var(X_1) + Var(X_2) + Var(X_3) = 3 \times 1.96 = 5.88;\) so \(\sigma_{(X_1+X_2+X_3)} = \sqrt{5.88} = 2.425.\)

13. a. \(E(X_1 + X_2 + \ldots + X_8) = E(X_1) + E(X_2) + \ldots + E(X_8) = 8 \times 1.7 = 13.6 \text{ calls}\)

b. \(Var(X_1 + X_2 + \ldots + X_8) = Var(X_1) + Var(X_2) + \ldots + Var(X_8) = 8 \times (0.81) = 6.48 \text{ calls}\)

14. a. \(E(X_1 + X_2) = E(X_1) + E(X_2) = \$27\)

\(Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 49 + 49 = 98\)

b. \(E(X_1 + X_2 + \ldots + X_{120}) = E(X_1) + E(X_2) + \ldots + E(X_{120}) = \$1620\)

\(Var(X_1 + X_2 + \ldots + X_{120}) = Var(X_1) + Var(X_2) + \ldots + Var(X_{120}) = 120 \times 49 = 5880 \text{ dollars}\)

28. geometric. a. \( p(4) = (0.95)^3(0.05) = 0.043 \); b. \( p(8) = (0.95)^7(0.05) = 0.0349 \); c. 
\[ E(X) = \frac{1}{0.05} = 20 \]

29. geometric; success = red candy; \( P(\text{success}) = 0.20 \);
\[ P(\text{first red is 4th candy selected}) = (0.80)^3(0.20) = 0.1024 \]

30. \[
1 + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 1 + \frac{6}{5} + \frac{6}{5} + \frac{6}{5} + \frac{6}{5} = 14.7
\]

Use the expected value of the geometric random variable. The first roll of a die necessarily gives us a number not rolled before. After we’ve rolled this first number (whatever it is), the probability we roll a different number is 5/6 since we want to roll one of the 5 numbers we haven’t yet rolled. The expected number of additional rolls until a second number turns up is thus 1/(5/6) or 6/5. After rolling two different numbers, the probability we roll a third number different from the first two is 4/6, and so the expected number of additional rolls until it turns up is 1/(4/6) or 6/4. Continue in this manner to obtain the answer shown above.

31. \[ P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.6288 = 0.3712 \]

32. a. the sampling distribution model is \( N(92, \frac{9}{\sqrt{81}}) \) b. \( P(z \leq -1.6) = 0.0548 \)

34. \[ P(\bar{x} < 47 \text{ or } \bar{x} > 53) = 0.1336 \]

35. \[ P(\bar{p} > 0.15) = P \left( \frac{\bar{p} - 0.12}{\frac{15}{\sqrt{80}}} > \frac{0.15 - 0.12}{\frac{15}{\sqrt{80}}} \right) = P(z > 1.07) = 0.142 \]

36. The sampling distribution model for the proportion of voters voting for Clinton is \( \hat{p} \sim N(.43, .0495) \)
\[ .95 = P(.43 - k \leq \hat{p} \leq .43 + k) = P \left( \frac{.43 - k}{.0495} \leq \frac{\hat{p} - .43}{.0495} \leq \frac{.43 + k}{.0495} \right) = P( -\frac{k}{.0495} \leq z \leq \frac{k}{.0495} ), \]
which implies that \( -\frac{k}{.0495} = -1.96, \frac{k}{.0495} = 1.96 \), so \( k = .097 \) and \( .43 - k = .333 \) and \( .43 + k = .527 \), so \( \hat{p} \) will have a value in the range (.333, .527) 95% of the time.