Review for Midterm Exam 2

1. Each box of Cracker Jack candied popcorn and peanuts contains 1 of 10 possible prizes. How many boxes of Cracker Jack do you expect to have to buy until you get all 10 prizes? Use the geometric rv. The first box you buy necessarily gives you a prize you don't yet have. After you have the first prize, when you buy another box the probability is \( p = \frac{9}{10} \) that you get a different prize, so the expected number of boxes until you get a different prize is \( \frac{1}{9/10} = \frac{10}{9} \). After you have 2 prizes, when you buy another box the probability is \( p = \frac{8}{10} \) that you get a different prize, so the expected number of boxes until you get a different prize is \( \frac{1}{8/10} = \frac{10}{8} \). And so on.

After you have 9 prizes, when you buy another box the probability is \( p = \frac{1}{10} \) that you get a different prize, so the expected number of boxes until you get a different prize is \( \frac{1}{1/10} = \frac{10}{1} \). So the expected number of boxes needed to get all 10 prizes is

\[
1 + \frac{10}{9} + \frac{10}{8} + \frac{10}{7} + \frac{10}{6} + \frac{10}{5} + \frac{10}{4} + \frac{10}{3} + \frac{10}{2} + \frac{10}{1} = 29.29
\]

2. You are the only bank teller on duty at your local bank. You need to run across the street for 10 minutes to Jimmy Johns to get a sub, but you don't want to miss any customers. Suppose the arrival of cutomers can be modeled by a Poisson distribution with mean 2 customers per hour.

a. What is the probability that no one will arrive in the next 10 minutes?

\[
\lambda = \frac{2}{6} = \frac{1}{3} \text{ customer per 10 minutes. } P(0) = \frac{e^{-(\frac{1}{3})(\frac{1}{6})}}{0!} = .717
\]

b. What is the probability that 2 or more people arrive in the next 10 minutes?

\[
P(X \geq 2) = 1 - P(X \leq 1) = .0446
\]

3. In a carnival game you can roll a fair 6-sided die repeatedly and add the number of dots showing on the upper face of each roll. You can stop at any time and receive an amount of money equal to the sum of all your rolls. However, if you roll a 5 your score is 0, the game is over, and you win nothing. Use the concept of expected value to devise a strategy for this game.
If you currently have \( P \) points and decide to roll again, there is a \( \frac{1}{6} \) chance for each of the following values after the next roll: \( P+1, P+2, P+3, P+4, 0, P+6 \). So the expected score after the next roll is

\[
(P+1)\frac{1}{6} + (P+2)\frac{1}{6} + (P+3)\frac{1}{6} + (P+4)\frac{1}{6} + 0 + (P+6)\frac{1}{6} = \frac{6P+16}{6}
\]

We would like this expected value to be greater than the current score of \( P \), that is, \( \frac{6P+16}{6} > P \), which implies \( P < 16 \).

So in the long run the optimal strategy is to stop rolling as soon as your total hits 16 or more.

(normal distribution)

4. The distribution of GPA's of undergraduates in the College of Science at NCSU can be approximated by a normal model with mean 3.12 and standard deviation 0.4. Suppose the College of Science has created a rigorous new Accelerated Science Studies Program to better prepare students to be globally competitive. A 3.72 GPA is required for a NCSU College of Science undergraduate to be accepted into the Accelerated Science Studies Program.

   a. What is the probability that a randomly selected NCSU College of Science undergraduate has a GPA of at least 3.72?

Let \( X \) denote the GPA of a NCSU College of Science undergrad.

\[
X \sim N(3.12, 0.4)
\]

\[
P(X \geq 3.72) = P\left( \frac{X - 3.12}{0.4} \geq \frac{3.72 - 3.12}{0.4} \right) = P(z \geq 1.5) = 0.0668
\]

   b. To attract high-quality current NCSU College of Science undergraduates into the new Accelerated Science Studies Program, scholarships will be offered to a NCSU College of Science undergraduate if his/her GPA is at or above the 95.54 percentile. What is the minimum GPA required to meet this criterion?

Use z-table or TI83/84 or Excel or Statcrunch.

Find \( k \) such that \( P(X \leq k) = .9554 \); from z-table \( P(z \leq 1.7) = .9554 \).

Therefore \( \frac{k - 3.12}{0.4} = 1.7 \Rightarrow k = 3.12 + 1.7 \times 0.4 = 3.8 \).