Confidence Intervals and Hypothesis Tests for $p_1 - p_2$

1. Attached is a poll from www.packpoll.com concerning opinions on gun control and concealed carry. The sample size is $n = 891$. How did packpoll determine the margin of error?

**Margin of error:** $1.96 \sqrt{\frac{(0.5)(0.5)}{891}} \approx 3.3\%$ (Packpoll reported 3.2%)

Comparing $p_1$ and $p_2$: independent samples

Assume $n_1 = 445$ responded to the first question Q1a (with non-assault-style Ruger rifle picture).

Define

$p_1 =$ proportion of people who would respond that they support a ban on semi-automatic weapons when shown picture of a non-assault style rifle with the question.

$\hat{p}_1 = .39 \ (x_1 = 173)$

Assume $n_2 = 446$ responded to the second question Q1a (with assault-style Ruger rifle picture).

Define

$p_2 =$ proportion of people who would respond that they support a ban on semi-automatic weapons when shown picture of an assault style rifle with the question.

$\hat{p}_2 = .44 \ (x_1 = 196)$

2. Calculate a 95% confidence interval to estimate $p_1 - p_2$.

$$n_1 = 445 \ x_1 = 173 \ \hat{p}_1 = \frac{173}{445} = .3888 \ n_2 = 446 \ x_2 = 196 \ \hat{p}_2 = \frac{196}{446} = .4395$$

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96 \ \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$= -.0508 \pm 1.96 \ \sqrt{\frac{.3888(.6112)}{445} + \frac{.4395(.5605)}{446}} = -.0508 \pm .0646 \rightarrow (-.1154, .0138)$

**NOTE:** $ti83/84$ gives $(-.1153, .0138)$

Since the interval is contains 0, it appears that $p_1$ is not significantly different than $p_2$. That is, showing the picture of an assault version of the rifle and a non-assault version of the rifle does not appear to have an affect on the proportion of people that respond that they support a ban on semi-automatic weapons.
3. Use the data to conduct the hypothesis test

\[ H_0 : p_1 - p_2 = 0 \]
\[ H_a : p_1 - p_2 \neq 0 \]

at significance level \( \alpha = 0.05 \), where

- \( p_1 \) = proportion of people who would respond that they support a ban on semi-automatic weapons when shown picture of a non-assault style rifle with the question
- \( p_2 \) = proportion of people who would respond that they support a ban on semi-automatic weapons when shown picture of an assault style rifle with the question

\[ \hat{p}_1 = \frac{173}{445} = 0.3888, \quad \hat{p}_2 = \frac{196}{446} = 0.4395; \quad \text{pooled} \ \hat{p} = \frac{173+196}{445+446} = 0.4141 \ldots \]

TEST STATISTIC:

\[ z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.3888 - 0.4395}{0.4141(1-0.4141)(\frac{1}{445} + \frac{1}{446})} = -1.536 \]

REJECTION REGION: \( z < -1.96 \) and \( z > 1.96 \)

CONCLUSION: Since the test statistic \( z \) is NOT in the rejection region, we do not reject the null hypothesis \( H_0 : p_1 - p_2 = 0 \); there is no evidence in the data that \( p_1 \) and \( p_2 \) are significantly different.

Note that \( P-value = 2P(z > |-1.536|) = 0.1245 > \alpha = 0.05 \)
These are the results of this semester’s first “Flash” Pack Poll, conducted through the internet, of a random sample of NCSU Freshman, Junior, and Masters Graduate students. The survey has a 34% response rate, and for the full sample a margin of error of +/- 3.2%.

For more information about the survey or further questions contact the Pack Poll at mike_cobb@ncsu.edu.

**Q1: Do you support or oppose a nationwide ban on semi-automatic weapons**

- Strongly Support……………………………….18%
- Support………………………………………   23%
- Oppose………………………………………..26%
- Strongly Oppose……………………….....31%

**Q1a: Do you support or oppose a nationwide ban on semi-automatic weapons**

- Support…………………………………………….39%
- Oppose………………………………………..61%

**Q1a: Do you support or oppose a nationwide ban on semi-automatic weapons (with assault-style Ruger Rifle picture)**

- Support…………………………………………….44%
- Oppose………………………………………..56%

**Q2: Do you support or oppose a law that would allow students who are 21 and older to carry concealed weapons on college campuses?**

- Strongly Support……………………………….17%
- Support………………………………………   24%
- Oppose………………………………………..28%
- Strongly Oppose……………………….....29%

**Q3: Do either of your parents own a gun?**

- Yes…………………………………………56%
- No…………………………………………39%
- I don’t know……………………………3%