Material covered on exam:

✓ Random variables, probability models, and sampling distributions in chapters 16, 17, and 18 (skip exponential models in chapter 17)

✓ webassign homework #6 through #8; worksheets 17 - 22.

Needed for exam:

✓ 8½" × 11" handwritten sheet of notes, formulas; calculator (I will supply needed tables)

1. A manufacturer of television sets has found that for the sets he produces, the lengths of time until the first repair can be described using a normal model with a mean of 4.5 years and a standard deviation of 1.5 years. If the manufacturer sets the warrantee so that only 10.2% of the 1st repairs are covered by the warrantee, how long should the warrantee last?

2. Suppose the amount of tar in cigarettes can be described using a normal model with a mean of 3.5 mg and a standard deviation of 0.5 mg.
   a. What is the probability that a cigarette has a tar content greater than 4.25 mg?
   b. In order to advertise as a low tar brand, a manufacturer must prove that their tar content is below the 25th percentile of the tar content distribution. Find the 25th percentile of the distribution of tar amounts.

3. The mean SAT verbal score of next year's freshmen entering the local university is 600. It is also known that 69.5% of these freshmen have scores that are less than 625. If the scores can be described using a normal model, what is the standard deviation of the scores?

4. Let $X$ be the number of accidents per week at a hazardous intersection; $X$ varies with mean 2.2 and standard deviation 1.4. $X$ takes takes only nonnegative whole-number values, so the distribution of $X$ is certainly not normal.
   a. Let $X_1$, $X_2$, and $X_3$ be the number of accidents in each of 3 different weeks at this intersection. The $X_i$s are independent and identically distributed, each with the same distribution as $X$. What is $SD(X_1 + X_2 + X_3)$, the standard deviation of the sum $X_1 + X_2 + X_3$?
   b. Let $\bar{x}$ be the mean number of accidents per week at the intersection during a year (52 weeks). What is the probability that $\bar{x}$ is less than 2?
   c. What is the probability that there are fewer than 100 accidents at the intersection in a year?

5. Determine which of the following functions is in fact a probability distribution function.
   a. $p(x) = \frac{1}{4}, x = 3, 4, 5, 6.$
   b. $p(x) = \frac{x^2}{25}, x = 0, 1, 2, 3, 4.$
   c. $p(x) = \frac{e^{-x^2}}{6}, x = 0, 1, 2, 3.$

6. A friend of yours plans to toss a fair coin 200 times. You watch the first 20 tosses and are surprised that she got 15 heads. But then you get bored and leave. How many heads do you expect her to have when she has finished all 200 tosses?
7. Assume the heights of high school basketball players can be described with a normal model. For boys the mean is 74 inches with a standard deviation of 4.5 inches, while girl players have a mean height of 70 inches and standard deviation 3 inches. At a mixed 2-on-2 tournament teams are formed by randomly pairing boys with girls as teammates (since the boy and girl teammates are chosen randomly, their heights are independent; the sum and difference of their heights can be described by a normal model).
   a. On average, how much taller do you expect the boy to be?
   b. What will be the standard deviation of the difference in teammates’ heights?
   c. On what fraction of the teams would you expect the girl to be taller than the boy?

8. A college student on a meal plan reports that the amount of money he spends daily on food varies with a mean (expected value) of $13.50 and a standard deviation of $7.
   a. Find the expected value and standard deviation of the amount he spends on 2 consecutive days (the amounts he spends on different days are independent).
   b. Find the expected value and standard deviation of the amount he spends during a semester that spans 120 days.

10. A certain hotel chain finds that 20% of the reservations made will not be used.
    a. What is the probability that among fifteen reservations made that more than 8 but less than 12 will be used?
    b. If four reservations are made, what is the probability that less than two will be unused?

11. The taste test for PTC (phenylthiourea) is a favorite exercise for every human genetics class. It has been established that a single gene determines the characteristic, and that 70% of the American population are "tasters," while 30% are "non-tasters". If 20 people are randomly chosen and administered the test:
    a. give the probability distribution of \( x \), the number of "non-tasters" out of the 20 chosen.
    b. Find \( P(3 < x < 9) \).
    c. Find the mean of \( x \).
    d. Find the variance of \( x \).

12. An oil firm is to drill twenty wells, with each well having probability 0.2 of successfully producing oil. It costs the firm $20,000 to drill each well. A successful well will bring in oil worth $750,000.
    a. Find the firm's expected gain \( G \) from the twenty wells.
    b. Find the variance of the firm's gain.

13. Ford claims that 40% of their customers are return buyers (i.e., they have purchased a Ford product previously). Suppose we sample 10 recent purchasers of a Ford product and define \( X = \# \) of return buyers.
    a. Find \( P(1 \leq x < 5) \).
    b. Find \( P(x > 6) \).
    c. What would you conclude about Ford's claim if you found 9 return buyers in the sample and why?

14. Identify the statement that describes a situation that can be modeled as a binomial experiment.
    a. a shopping mall is interested in the income levels of its customers and is taking a survey to gather information
    b. a business firm introducing a new product wants to know how many purchases its clients will make each year
    c. a sociologist is researching an area in an effort to determine the proportion of households with male "heads of household"
    d. a study is concerned with the average hours worked by teenagers who are attending high school

15. Which of the following is a correct statement concerning the Central Limit Theorem (CLT)?
    a) The CLT states that the sample mean, \( \bar{x} \), is always equal to \( \mu \).
b) The CLT states that for large samples, the sampling distribution of the sample mean is approximately normal.

c) The CLT states that for large samples, sample mean \( \bar{x} \) is equal to \( \mu \).

d) The CLT states that for large samples, the sampling distribution of the population mean is approximately normal.

e) Both c and d are correct.

16. The amount of money spent on food per week by a typical American family is known to have a mean of 92 dollars with a standard deviation \( \sigma \) of 9 dollars. Suppose a random sample of 81 families is taken and the sample mean is calculated.

a. Describe the sampling distribution model of the sample mean. (Include the mean, standard deviation, and type of distribution if known).
b. Find the probability that the sample mean does not exceed 90.4 dollars.

17. In a learning experiment, untrained mice are placed in a maze and the time required for each mouse to exit the maze is recorded. For untrained mice, the average time to exit the maze is \( \mu = 50 \) sec and the standard deviation is \( \sigma = 16 \) sec. If 64 randomly selected untrained mice are placed in the maze and the time necessary to exit the maze recorded for each one, what is the probability that the sample mean differs from 50 by more than 3?

18. During the lunch-hour rush at a McDougalds restaurant orders for the Big Mac hamburger follow a Poisson distribution and occur at a rate of 4 per minute. What is the probability that 5 or more Big Macs will be ordered in one minute?

19. In the 1992 U.S. presidential election, Bill Clinton received 43% of the vote compared to 38% for George H. W. Bush and 19% for Ross Perot. Suppose we had taken a random sample of 100 voters in an exit poll and asked them for whom they had voted. In 95% of such polls, our sample proportion of voters for Clinton should be between what two values that are equidistant from the expected value?

20. The owner of a small convenience store notices that only 5% of customers buy magazines.

a. What is the probability that the first customer to buy a magazine is the 4th customer?
b. What is the probability that the first customer to buy a magazine is the 8th customer?
c. How many customers should the owner expect until a customer buys a magazine?

**SOLUTIONS**

1. \( z = -1.27 ; \ x = 2.595 \) years.

2. \( a. 0.668 \quad b. z = -0.675 \quad Q_1 = 3.16. \quad 3. 0.51 = (625-600)/\sigma \Rightarrow \sigma = 49.02. \quad 4. \text{a. Since the } X_i \text{'s are independent,} \quad Var(X_1 + X_2 + X_3) = Var(X_1) + Var(X_2) + Var(X_3) = 5.88 \text{; so} \quad SD(X_1 + X_2 + X_3) = \sqrt{5.88} = 2.425. \quad \text{b. By the CLT, } \pi \sim \text{approx. } N(2.2, .194), \text{ so} \quad P(\pi < 2) = P\left(\frac{\pi - 2.2}{0.194} < -1.03\right) = P(z < -1.03) = .1515. \quad \text{c. } P(\text{total accidents} < 100) = P(\pi < \frac{100}{2.2}) = P(\pi < 1.923) = P(z < -1.43) = .0764.\)

5. a. is the only legitimate prob. distribution since in b. \( \sum p(x) > 1 \); in c. \( p(3) < 0. \)

6. 105

7. \( a. \text{Let } B - G \text{ denote the boy's height minus the girl's height; } E(B - G) = E(B) - E(G) = 74 - 70 = 4 \) inches; \( b. \text{Var}(B - G) = \text{Var}(B) + \text{Var}(G) = (4.5)^2 + (3)^2 = 29.25. \text{SD}(B - G) = \sqrt{29.25} = 5.4; \text{ c. if the girl is taller than the boy, then } B - G < 0. \text{ P}(B - G < 0) = P\left(B < -\frac{G}{5.4}\right) = P(z < -0.74). \) From the standard normal table the proportion of the area to the left of \( -0.74 \) is 0.2296.

8. \( a. E(X_1 + X_2) = E(X_1) + E(X_2) = 27 \quad Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 49 + 49 = 98 \quad \text{std dev} = \sqrt{98} = 9.899 \)
b. Let \( X_i \) = the amount he spends on food on day \( i \), \( i = 1, 2, 3, \ldots, 120 \)

\[
E(X_1 + X_2 + \ldots + X_{120}) = E(X_1) + E(X_2) + \ldots + E(X_{120}) = 1620 \\
Var(X_1 + X_2 + \ldots + X_{120}) = Var(X_1) + Var(X_2) + \ldots + Var(X_{120}) = 120 \times 49 = 5880 \\
\text{stan dev} = \sqrt{5880} = 76.68
\]

10. a. \( p = 0.8 \); \( P(8 < x < 12) = 0.334 \)  
   b. \( p = 0.2 \); \( P(x < 2) = 0.8192 \)

11. a. \( p(x) = \frac{20!}{x!(20-x)!} (0.3)^x (0.7)^{20-x} \) \( x = 0, 1, 2, \ldots, 20 \)  
   b. 0.78  
   c. 6  
   d. 4.2

12. Let \( X = \# \) of successful wells; \( X \sim \text{binomial}(20, 0.2) \). \( \text{Gain} = 750,000(X) - 20(20,000) \)  
   a. \( E(G) = 750,000E(X) - 400,000 = 750,000(4) - 400,000 = 2,600,000 \)  
   b. \( \text{Var}(G) = (750,000)^2 \text{Var}(X) = (750,000)^2(20)(0.2)(0.8) \)

13. a. 0.627  
   b. .055  
   c. Have reason to doubt Ford's claim since the probability that \( x = 9 \) is approximately .002 when \( p = .4 \).

14. c.  
   15. b.  
   16. a. the sampling distribution model is \( N(92, \frac{9}{\sqrt{91}}) \)  
   b. \( P(z \leq -1.6) = 0.0548 \)

17. \( P(\bar{X} < 47 \text{ or } \bar{X} > 53) = 0.1336 \)

18. \( P(X \geq 5) = 1 - P(X \leq 4) = 1 - .6288 = .3712 \)

19. The sampling distribution model for the proportion of voters voting for Clinton is \( \hat{p} \sim N(.43, .0495) \)  
   \[ .95 = P(.43 - k \leq \hat{p} \leq .43 + k) = P\left(\frac{.43 - k}{.0495} \leq \frac{\hat{p} - .43}{.0495} \leq \frac{.43 + k}{.0495}\right) = P\left(\frac{-k}{.0495} \leq z \leq \frac{k}{.0495}\right), \]
   which implies that \( \frac{-k}{.0495} = -1.645 \), \( \frac{k}{.0495} = 1.645 \), so \( k = .97 \) and \( .43 - k = .333 \) and \( .43 + k = .527 \), so \( \hat{p} \) will have a value in the range \((.333, .527)\) 95% of the time.

20. geometric. a. \( p(4) = (0.95)^4(0.05) = 0.043 \);  
   b. \( p(8) = (0.95)^7(0.05) = 0.0349 \);  
   c. \( E(X) = \frac{1}{0.05} = 20 \)