#Q10.8
INTRATE30=read.table(file.choose(),header=TRUE)
TIME <- INTRATE30$Year - 1985
out <- lm(INTRATE30$IntRate ~ (TIME))
#prediction at t=25
newdata=data.frame(TIME=25)
predict(out,newdata,interval="predict")

(a)
Call:
lm(formula = INTRATE30$IntRate ~ (TIME))

Coefficients:
(Intercept)         TIME
11.3182         -0.2733

Regression equation is
IntRate=11.3182-.2733 t

(b)

<table>
<thead>
<tr>
<th>fit</th>
<th>lwr</th>
<th>upr</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.485395</td>
<td>2.437796</td>
<td>6.532995</td>
</tr>
</tbody>
</table>

The fitted value in 2010 is $y\text{-hat}=11.3182-.2733\times25=4.485$, with 95% prediction lower bound and upper bound 2.43 and 6.53 respectively.

#Q10.26 continued from above
acf(residuals(out))

Series residuals(out)

![ACF plot](image)
Consider first order Autoregressive modelling,

```
ar1<-arima(INTRATE30$IntRate,order=c(1,0,0),xreg=TIME)
print(ar1)
```

```
arima(x = INTRATE30$IntRate, order = c(1, 0, 0), xreg = TIME)
```

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>intercept</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3293</td>
<td>11.3183</td>
<td>-0.2703</td>
</tr>
</tbody>
</table>

s.e. 0.2015 0.4572 0.0353

sigma^2 estimated as 0.6388: log likelihood = -27.54, aic = 63.08

\[ y_t = \beta_0 + \beta_1 t + \varphi R_{t-1} + \varepsilon_t \]

\[ \hat{\beta}_0 = 11.3183, \quad \hat{\beta}_1 = -0.2703, \hat{\varphi} = 0.3293 \]

The intercept estimate is the predicted rate for the base year 1985, beta_1 estimate is the estimated decrease in the interest rate due to pure time trend as each year goes by after 1985, the \( \varphi \) is the autoregressive constant.

A test about the \( H_0: \varphi = 0 \), yiled t statistic = \( 0.3293/0.2015 \), with 21 degrees of freedom is not significant at 5% level. Hence the first order autoregressive model does not describe the residuals correlation well.

**10.32**

From previous table,

Coefficients:

<table>
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</table>

s.e. 0.2015 0.4572 0.0353

sigma^2 estimated as 0.6388: log likelihood = -27.54, aic = 63.08

Note the observations range from n=1,2,3,...23,

\[ F_{24} = \hat{\beta}_0 + \hat{\beta}_1 (23 + 1) + \hat{\varphi} \hat{R}_{23} \]

where \( \hat{R}_{23} = Y_{23} - \hat{Y}_{23} = 6.4 - (11.3182 - 0.2703 \times 23) = 1.29 \)

So

\[ F_{24} = \hat{\beta}_0 + \hat{\beta}_1 (23 + 1) + \hat{\varphi} \hat{R}_{23} = 11.3182 - 0.2703 \times 24 + 0.3292 \times 1.2987 = 5.258 \]
95% One-step ahead forecast limits:

\[ F_{24} \pm 2\sqrt{\text{MSE}} \], where MSE is the variance of the uncorrelated \( \varepsilon_t \), which is equal to 0.6388. So

\[ 5.258 \pm 2\sqrt{0.6388} \] gives ( 4.46, 6.06 )