Exercises 5.18, 5.20, 5.26, 5.36

5.18
a.  \[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 + \epsilon \]

b. 14% of the variability in the goal of improving efficiency is explained by the level of CEO leadership and the level of congruence between the CEO and the VP.

c. This could show that the relationship between the CEO and VP has a negative effect on the efficiency of the company; i.e. if the CEO an VP do not “get along” in the eye business eye, the efficiency of the company could suffer.

d. Since \( p\text{-value} = .02 < \alpha = .05 \), this implies that the interaction between the CEO leadership and the CEO and VP congruence has a significant impact on helping to predict the variability in the goal of improving efficiency.

5.20
a. We use the following summary information to calculate and \( s_x \) for the sample of \( x \)-values:
   \[ n = 7, \quad \sum x^2 = 7651, \quad \sum x = 231 \]
   Then \( \bar{x} = \frac{\sum x}{n} = \frac{231}{7} = 33 \)
   \[ s_x = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n-1}} = \sqrt{\frac{7651 - (231)^2}{6}} = \sqrt{4.667} = 2.16 \]

b. Using the coding system for observational data, we have:

\[
\begin{array}{cccccc}
& 30 & 31 & 32 & 33 & 34 & 35 & 36 \\
\hline
u & -1.329 & -0.926 & -0.463 & 0 & 0.463 & 0.926 & 1.389 \\
\end{array}
\]

c. Let \( x_1 = x \) and \( x_2 = x^2 \). We first calculate:
   \[ \sum x = 231 \quad \sum x^2 = 7651 \]
   \[ \sum x_1^2 = 7651 \quad SS_{x_1 x_1} = 28 \quad \sum x_2^2 = 8484595 \quad SS_{x_2 x_2} = 1848 \]
   \[ \sum x_1 x_2 = 254331 \quad SS_{x_1 x_2} = 122052 \]
   Then, \( r = \frac{SS_{x_1 x_2}}{\sqrt{SS_{x_1 x_1} SS_{x_2 x_2}}} = \frac{1848}{\sqrt{(28)(122,052)}} = .9997 \)
d. Let $u_1 = u$ and $u_2 = u^2$. We first calculate:

\[
\begin{align*}
\sum u_1 &= 0 & \sum u_2 &= 6.002 & \sum u_1^2 &= 6.002 & SS_{u_1} &= 6.002 & \sum u_2^2 &= 9.007 \\
SS_{u_1 u_2} &= 0 & \sum u_1 u_2 &= 0 & SS_{u_2} &= 3.860
\end{align*}
\]

Then, \( r = -\frac{SS_{u_1 u_2}}{\sqrt{SS_{u_1} SS_{u_2}}} = 0 \)

\[
\frac{0}{\sqrt{(6.002)(3.860)}} = 0
\]

e. Using the SAS multiple regression procedure, we obtain:

\[
\hat{y} = 37.5714 - 0.4629u - 5.3333u^2
\]

5.26 a. Group is the qualitative independent variable. It must be coded into two dummy variables since it has three levels.

b. \( E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \) where \( x_1 = \begin{cases} 
1 & \text{if group 2} \\
0 & \text{otherwise}
\end{cases} \quad x_2 = \begin{cases} 
1 & \text{if group 3} \\
0 & \text{otherwise}
\end{cases} \)

c. \( \beta_0 = \mu_1 \) mean milk production of cows in group 1 (man-made shade structures)

\( \beta_1 = \mu_2 - \mu_1 \) difference in mean milk production between cows in group 2 and group 1 (tree shade minus man-made shade structure)

\( \beta_2 = \mu_3 - \mu_1 \) difference in mean milk production between cows in group 3 and group 1 (no shade minus man-made shade structure)

5.36 a. \( E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_1 x_2 + \beta_5 x_1^2 x_2 \)

b. For non-coached students: \( x_2 = 0 \)

\[
E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 (0) + \beta_4 x_1 (0) + \beta_5 x_1^2 (0)
\]

y-intercept: \( \beta_0 \)
shift parameter: \( \beta_1 \)
r rate of curvature: \( \beta_2 \)
c. For coached students: \( x_2 = 1 \)

\[
E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 (1) + \beta_4 x_1 (1) + \beta_5 x_1^2 (1)
\]

\[
= (\beta_0 + \beta_3) + (\beta_1 + \beta_4) x_1 + (\beta_2 + \beta_5) x_1^2
\]

y-intercept: \( \beta_0 + \beta_3 \)
shift parameter: \( \beta_1 + \beta_4 \)
rate of curvature: \( \beta_2 + \beta_5 \)

d. To determine if coaching has an effect on SAT-Math, we test:

\[
H_0: \beta_3 = \beta_4 = \beta_5 = 0
\]

\[
H_a: \text{At least one } \beta_i \neq 0
\]