Statistics 435/535

Statistical Methods for Quality and Productivity Improvement / Statistical Process Control

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Course home page:
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The Meaning of Quality and Quality Improvement

Dimensions of quality for products

- **Performance:** Will the product do the intended job?
- **Reliability:** How often does the product fail?
- **Durability:** How long does the product last?
- **Serviceability:** How easy is it to repair the product?
- **Aesthetics:** What does the product look like?
- **Perceived quality:** What is the reputation of the company or its product?
- **Conformance to standards:** Is the product made exactly as the designer intended?
Dimensions of quality for services

- **Responsiveness**: How long did it take the service provider to reply to the request for service?
- **Professionalism**: Is the organization competent to provide the requested service?
- **Attentiveness**: Was the service provided with caring and personalized attention?
Traditional definition of quality

Products and services must meet the needs of those who use them:

*Quality means fitness for use.*

- Fitness for use depends on the original design and on the product’s conformance to that design.
- Too much focus on conformance leads to less focus on the customer.
Modern definition of quality

Products and services must be reliable and predictable:

\textit{Quality is inversely related to variability.}


- Same transmission built in the US and in Japan.
- Warranty claims and repair costs higher for US-built transmissions.
- Random samples of US-built and Japan-built units showed more variability in critical dimensions in US-built units.
- Japan-built transmissions shifted more smoothly, ran more quietly, and were perceived as superior.
Quality Improvement

*Quality improvement is the reduction of variability in processes and products.*

- Reducing variability leads to increased perception of quality.
- Reducing variability also leads to reduction in waste and the associated costs.
Terminology

A **quality characteristic** (critical-to-quality, CTQ) is some aspect of a product or service that influences its perceived quality.

- E.g. the diameter of a shaft in an engine, or the time taken to handle the return of a purchase.
- We shall be concerned mostly with quality characteristics that may be *measured*, unlike say taste or smell.

A quantitative quality characteristic typically has a **nominal** or **target** value.

Characteristics typically deviate at least a little from the nominal value. The largest acceptable deviations are called the **upper specification limit** (USL) and **lower specification limit** (LSL).
Product example

The main bearings of a 1989 5.9 litre Dodge Ram truck engine had a nominal diameter of 2.81 inches, with USL 2.8105 and LSL 2.8095. A shaft with diameter outside these limits is nonconforming.

- If the shaft is too large, it will bind and overheat.
- If the shaft is too small, it will cause noise and low oil pressure.

Service example

Zappos promises to issue a refund within 7 days of receiving and inspecting a return. That is the USL: we don’t know what their internal target is. There is no LSL: faster is always better.

- If the refund takes longer than 7 days, the customer is inconvenienced, and less likely to buy from Zappos.
Statistical Methods for Quality Control and Improvement

Many tools for quality control and improvement are management-related, not statistical. Eg. quality circles, 14-point plans, and so on. These are outside the scope of this course.

Major statistical areas

- Statistical process control
- Design of experiments
- Acceptance sampling
Statistical process control

Example: the control chart:

- Measure a quality characteristic in a sample from the process;
- Examples:
  - Diameters of engine shafts sampled from the production line;
  - Refund times for one day;
- Plot against time to look for changes.

![Control chart example](image)
Design of experiments (DoE)

Designed experiments are used to find the key factors that influence a quality characteristic, and to characterize their effects.

DoE is often used off-line; that is, not as part of the routine operation of the process.

Factorial designs use all combinations of chosen levels of the factors of interest.

At an early stage, many factors may be screened for effects, and a fractional factorial design may be used.
Acceptance sampling:
A tool used in managing *product* quality.

Example
A manufacturer ships 45oz boxes of detergent to a retailer. The contract specifies that no more than 2% of boxes should have contents outside the specification limits. The retailer inspects a sample of boxes from a shipment on arrival, and if too many are nonconformant, the whole shipment is returned to the manufacturer.

- How many boxes should be sampled?
- How many nonconformant boxes should be allowed?

Note
The manufacturer has an incentive to improve quality by reducing variability, to reduce the cost of returned shipments.
Management Aspects of Quality Improvement

Six Sigma

Six Sigma is a set of techniques and tools for process improvement, developed by Motorola in the 1980’s.

Suppose that some quality characteristic has a target value $T$ and specification limits $T \pm L$.

Suppose also that the system results in values $X$ that are normally distributed (unlikely!) with mean $\mu$ and standard deviation $\sigma$.

Then the probability that the product meets the specs is

$$P(T - L \leq X \leq T + L) = \Phi\left(\frac{T + L - \mu}{\sigma}\right) - \Phi\left(\frac{T - L - \mu}{\sigma}\right).$$
If \( T = \mu \) (that is, on average the characteristic is on target) and \( L = 3\sigma \) (Three Sigma quality performance) then the probability of being in spec is 0.9973.

That is, 99.73% of values are in spec, or 2700 in a million are out of spec.

But if a product contains 100 such parts, statistically independent, then the probability that all 100 parts are in spec is

\[
(0.9973)^{100} = 0.7631.
\]

That is, the product has a 23.7% chance of containing at least one part that is out of spec.
R illustration

curve(dnorm(x, sd = 1/3), from = -1.2, to = 1.2,
    axes = FALSE, xlab = "", ylab = "");
box();
abline(h = 0);
abline(v = 0, col = "green");
abline(v = c(-1, 1), col = "red");
title(paste("P(out of spec) =",
    signif(1 - (pnorm(3) - pnorm(-3)))));

Note

The semi-colon at the end of each command is redundant: the end of a line marks the end of the command. However, when several lines are copied and pasted into an R window, they may be run together into a single line, and in that case the semicolon is necessary to separate the commands.
R illustration, continued

```r
cat("P(out of spec) =",
    signif(1 - (pnorm(3) - pnorm(-3))), "\n")
cat("P(any of 100 out of spec) =",
    signif(1 - (pnorm(3) - pnorm(-3))^100), "\n")
```

Motorola

Reduce variability $\sigma$ until $L = 6\sigma$ (Six Sigma quality). Then the probability of a single part being out of spec is $1.97 \times 10^{-9}$.

The probability of 100 independent parts all being in spec is $(1 - 1.97 \times 10^{-9})^{100} = 1 - 1.97 \times 10^{-7}$, so now the 100-part product has only a 0.0000197% chance of containing any defective parts.
Change in mean

In illustrations like these, the mean $\mu$ is often assumed to be off-target by up to $1.5\sigma$:

$$T - 1.5\sigma \leq \mu \leq T + 1.5\sigma.$$ 

Then the probability of a single part being out of spec is at most $3.40 \times 10^{-6}$, or 3.4 defects per million parts.

The probability of 100 independent parts all being in spec is at least $(1 - 3.40 \times 10^{-6})^{100} = 1 - 3.40 \times 10^{-4}$, so the 100-part product has at most a 0.034% chance of containing any defective parts.
“Six Sigma” is used more broadly to cover activities aimed at reducing the percentage of out-of-spec parts or services to similar low levels, not necessarily as low as 3.4 per million.

**Note**

All the calculations above assume that the quality characteristic is normally distributed.

If the distribution is $t$ with 10 degrees of freedom:

- If the distribution is centered, the out-of-spec rate is 53 per million;
- If the mean shifts by up to $1.5\sigma$, the out-of-spec rate is up to 260 per million.

But these two distributions are almost indistinguishable.