1. Here is the $X$ matrix and $X'X$ from a regression problem.

\[
X = \begin{pmatrix}
1 & 0 & 2 \\
1 & -2 & 1 \\
1 & 3 & 0 \\
1 & 1 & 1 \\
1 & 2 & 0 \\
1 & 0 & 4 \\
1 & -1 & 1
\end{pmatrix}
\quad
X'X = \begin{pmatrix}
\_ & 3 & \_ \\
3 & \_ & \_ \\
9 & \_ & 23
\end{pmatrix}
\]

The corresponding estimated regression equation is

\[\hat{Y} = 10 + 3X_1 + 2X_2\]

where, as usual, $\hat{Y}$ denotes the predicted value of $Y$. The error mean square is 9.

(A) (10 pts.) What is the predicted value of $Y$ when $X_1 = 3$ and $X_2 = 4$?

(B) (30 pts.) Fill in the missing $X'X$ entries.

(C) (6 pts.) How many rows ___ and columns ___ does $X'Y$ have?

(D) (6 pts.) Compute, if possible from the given information, $\bar{Y}$ = _____ (the sample average $Y$ value)

(E) (6 pts.) How many error degrees of freedom _____ do we have in this regression?

(F) (10 pts.) Find, if possible, $(X'X)^{-1}(X'Y) = \begin{pmatrix}
\_
\end{pmatrix}$. If not possible, put “NP”.

(G) (6 pts.) Suppose I want to test the null hypothesis $H_0 : \beta_1 = 0$ where $\beta_1$ is the coefficient of $X_1$. How many degrees of freedom ____ would I have for my t test?

(H) (8 pts.) There is a population of $Y$ values with $X_1 = 3$ and $X_2 = 4$. We assume that population is normally distributed.

Estimate the mean ____ and variance ____ of that population.
2. Some matrix questions:

(A) (6 pts.) Compute the matrix sum
\[
\begin{pmatrix}
0 & 5 \\
-3 & 2
\end{pmatrix} + \begin{pmatrix}
2 & 0 \\
4 & -1
\end{pmatrix} = \begin{pmatrix} \_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_
\end{pmatrix}
\]

(B) (12 pts.) Find a lower right corner element \( x \) for this matrix
\[
\begin{pmatrix}
1 & 3 & 5 \\
1 & 3 & x
\end{pmatrix}
\]
such that

(i) the matrix has an inverse \( x = \_ \)

(ii) the matrix has rank 2. \( x = \_ \)

Note there may be one or several correct answers, or none (if none, put “NP”).

\[10 + 3(3) + 2(4) = 27 \text{ This is the estimated mean } Y \text{ at these } X \text{ values (needed later)}\]

Once we get the entries on the diagonal an below, symmetry gives those above the diagonal – no need to compute.

\[
\begin{pmatrix}
1 & 0 & 2 \\
1 & -2 & 1 \\
1 & 3 & 0 \\
1 & 1 & 1 \\
1 & 2 & 0 \\
1 & 0 & 4 \\
1 & -1 & 1
\end{pmatrix}
\]

\[
X'X = \begin{pmatrix}
-7 & 3 & -9 \\
3 & -19 & -2 \\
9 & -2 & 23
\end{pmatrix}
\]

3.1
This takes some thought. Note that \( X'Y \) has the sum of the \( Y \) values as the first entry as we have mentioned several times. Also the important NORMAL EQUATIONS are

\( X'Xb = X'Y \) and since we know \( b \) and \( X'X \) we can get the first entry in \( X'Xb \) from the first row, first column entry of this product, namely \( (7 \ 3 \ 9)b = 7(10) + 3(3) + 9(2) \) where the coefficients 10, 3, 2 are the entries in the column vector \( b \) and \( (7 \ 3 \ 9) \) is the first row in \( X'X \).

\[n=7 \text{ so } 7-3=4 \text{ df.}\]
This is just the formula (solution to the normal equations) for the \( \mathbf{b} \) vector so entries are 10, 3, 2

Test also has 4 df – same as error term.
27 (see A) and MSE = 9.

2. Sum is \[
\begin{pmatrix}
2 & 5 \\
1 & 1 \\
\end{pmatrix}
\]. First 2 columns are dependent (second is direct multiple of first)

If \( x \) matches the numbers above it, rank is 1. ANY other \( x \) gives rank 2. No \( x \) gives full rank so no \( x \) produces invertible matrix.