1. Here is a table of (P, M) totals for a 2x3x2 experiment with factors N (nitrogen), P (pH), and M (moisture) at 2, 3, and 2 levels. I have also given what we called the “table sum of squares” in our notes. The experiment was a randomized complete block design with 6 blocks.

<table>
<thead>
<tr>
<th></th>
<th>P 6.5</th>
<th>P 7.0</th>
<th>P 7.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 20</td>
<td>714</td>
<td>840</td>
<td>1260</td>
</tr>
<tr>
<td>M 40</td>
<td>756</td>
<td>1260</td>
<td>1470</td>
</tr>
<tr>
<td>Totals</td>
<td>1470</td>
<td>2100</td>
<td>2730</td>
</tr>
</tbody>
</table>

Table SSq = \( \frac{714^2}{12} + \frac{840^2}{12} + \ldots + \frac{1470^2}{12} - \frac{6300^2}{72} = 42336 \)

A) (10 pts.) How many observations _____ do I have in all?

B) (24 pts.) Compute, if possible, the missing sums of squares and degrees of freedom for this part of the analysis of variance table. If not possible, explain.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Sum of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>6272</td>
</tr>
<tr>
<td>P</td>
<td>___</td>
<td>______________</td>
</tr>
<tr>
<td>MP</td>
<td>___</td>
<td>______________</td>
</tr>
</tbody>
</table>

C) (8 pts.) Using the orthogonal polynomial coefficients 1, -2, 1 for quadratic effects, compute the sum of squares ______ for testing to see if the quadratic P effect is the same at both levels of M.

2. I have a data set with 21 points (X,Y) but there are only 7 distinct values of X. Y is yield and X is temperature. I ran this code:

```plaintext
PROC GLM; CLASS X; MODEL Y=X;
```

Obtaining this analysis of variance (without sums of squares):

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>Source</th>
<th>df</th>
<th>Sum of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>___</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>___</td>
<td>______________</td>
</tr>
</tbody>
</table>

A) (15 pts.) Fill in the missing degrees of freedom (df) and error mean square.

B) (5 pts.) Do I accept or reject the null hypothesis of no temperature effect on yield at the usual 5% significance level?

C) (10 pts.) If we fit a quadratic (degree 2) polynomial model to the data, how many degrees of freedom _____ would we have for lack of fit of this quadratic model?
3. (28 pts.) I have a 2x2 factorial with factors A and B and with 7 replicates in blocks. The overall mean of all the data is 120. The main effect of A is 10 and the main effect of B is -6. Compute if possible (if not, put “NP”)
(A) The block degrees of freedom______
(B) The error degrees of freedom_______
(C) The mean _____ of all the data with A at the low level.
(D) The sum of squares _______ for the A main effect.

1. (A) Each table number is summed over 6 reps and 2 levels of N. Thus n = 6(12) = 72.
(B) Using the column totals (each a total of 24 observations)
   \[ SS(P) = \frac{1470 	imes 1470}{24} + \ldots + \frac{2730 	imes 2730}{24} = CT = 33075 \text{ with } 3-1=2 \text{ df} \]
   By subtraction \[ SS(MP) = 42336 - 6272-33075 = 2989 \text{ with } 2 \text{ df} \]
   (C) Applying coefficients \[ 1 \quad -2 \quad 1 \]
   to the top row and \[ -1 \quad 2 \quad -1 \]
   to the bottom totals in our table we have \[ Q = 588 \text{ and denominator } (1+4+1+1+4+1)(12) = 144 \text{ so } \frac{588 	imes 588}{144} = 2401. \]

2. (A) X has 7 levels so 6 df when used in a class statement. 21 points total so 20 total df leaving 20-6=14 for error. \[ F = \frac{MS(X)}{MSE} \text{ so } 4 = \frac{800}{MSE} \text{ and MSE must be } 200. \]
   (B) The p-value is 0.0153 so I reject my null hypothesis (no effect of temperature X on response Y) at the 5% level. In other words it is unlikely by that criterion to see such an F if there really is no effect.
   (C) A quadratic has 2 df, so 6 df for X, and 2 df for the quadratic leaves 6-2 = 4 for lack of fit. You could also reason that we could fit up to a degree 6 polynomial so the omitted powers of X for a quadratic model are 3,4,5,6, that is, there are 4 powers of X that could have been included but weren’t.

3. 6 df for blocks and 18 for error
   (also 3 df for 4 treatments, 4(7)=28 observations, 27 total df)
   (C) The high level average minus the low level is the main effect. The overall average is 120 so one of those means must be 5 above and one must be 5 below 120. Since the main effect is positive, the high level mean must be larger and the low level mean must be smaller so these means are 115 (the answer) and 125.
   (D) For the A main effect I need totals. These would be 115(14) and 125(14) so \[ Q = 10(14) \text{ which of course could be quickly computed as } 14 \text{ times the main effect } 10. \]
   We have \[ \frac{(140)(140)}{28} = 700 \text{ as the sum of squares. You could also square the two totals, divide each square by } 14, \text{ add them up and subtract the correction term.} \]