1. (20 pts) Suppose the hazard function of a random survival time (in years) $T$ takes the form 

$$
\lambda(t) = \left(\frac{\lambda}{2}\right)t^{-1/2},
$$

where $\lambda > 0$. Do the following:

(a) Find the cumulative hazard function, survival function and density function of $T$. (8 pts)

(b) Given a random sample $(\tilde{t}_1, \delta_1), (\tilde{t}_2, \delta_2), \ldots, (\tilde{t}_n, \delta_n)$, where $\tilde{t}_i$ is the observed failure time or right censoring time; $\delta_i = 1$ if $\tilde{t}_i$ is a failure time and $\delta_i = 0$ if $\tilde{t}_i$ is a censoring time. Find the MLE of $\lambda$ and its asymptotic variance. (8 pts)

(c) Suppose we have a sample $1, 4+, 4, 9$, where $+$ indicates a right censored observation. Find the MLE of $\lambda$ and its variance. (4 pts)

2. (20 pts) The following table gives grouped survival data in years for patients with certain disease:

<table>
<thead>
<tr>
<th>time interval $i$</th>
<th>$n_i$</th>
<th>$d_i$</th>
<th>$w_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$: [0, 2)</td>
<td>20</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$i = 2$: [2, 4)</td>
<td>16</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$i = 3$: [4, 6)</td>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

where $n_i$ is the number of patients at the beginning of time interval $i$, $d_i$ and $w_i$ are the number of deaths and censorings observed in that interval. Assume all censorings in an interval occurred at the middle of that interval. Do the following:

(a) Find the life-table estimate of $S(6) = P(T \geq 6)$. (10 pts)

(b) Find the variance estimate for the estimate you got in (a). (10 pts)

3. (40 pts) The survival times of 10 patients with certain heart disease are given as follows:

$$
2, 3, 4+, 5, 5+, 6+, 8, 10, 12, 15+,
$$

where “+” means a right censored survival time, do the following:

(a) Find the Nelson-Aalen estimator of the cumulative hazard $\Lambda(8)$ and the Kaplan-Meier estimator of $S(8) = P(T > 8)$. (15 pts)
(b) Find the variance estimates for the Nelson-Aalen and Kaplan-Meier estimators you got in (a). (15 pts)

(c) Construct a 95% confidence interval for $S(8)$ based on the Nelson-Aalen and Kaplan-Meier estimators you got in (a). (10 pts)

4. (20 pts) In a clinical trial, we want to compare a new treatment to the standard one on time (in months) to tumor recurrence for patients with certain type of cancer. Assume that enough patients will be available at the beginning of the trial and half of them will be randomly assigned to each treatment. It is known that the average tumor recurrence time is 2 month for the standard treatment; while for the new treatment, it is expected that it will extend the average tumor recurrence time to 4 month. The trial will last 10 months. Suppose that the tumor recurrence time for each treatment group follows an exponential distribution and the censoring time is uniformly distributed on the interval $[0, 10]$. Do the following:

(a) Find the probability for each treatment that a patient will experience the tumor recurrence before the end of the trial. (10 pts)

(b) Suppose we will use the log-rank test to test the difference in time to tumor recurrence for these two treatments with significance level $\alpha = 0.05$ and 99% power to detect the expected difference. How many patients should be included in the study? (10 pts)