ST512

Fall Quarter, 2005

Exam 3

Name:__________________________________________________________

Directions: Answer questions as directed. Show work where appropriate. For true/false questions, circle either true or false.
1. (40 pts) 25 men are randomly sampled from a population of interest and diastolic blood pressures is measured on each of 16 occasions. Let $y_{ij}$ denote the measurement for subject $i$ and occasion $j$. Summary statistics are given below.

$$
\bar{y}_{++} = 91.7, \quad \sum_{i=1}^{25} \sum_{j=1}^{16} (y_{ij} - 91.7)^2 = 11529, \quad \sum_{i} \sum_{j} (y_{ij} - \bar{y}_{i+})^2 = 19541
$$

Consider a model with subject effects $A_i$ assumed i.i.d. $N(0, \sigma_1^2)$ and errors $E_{ij}$ assumed i.i.d. $N(0, \sigma_2^2)$, independent of $\{A_i\}$.

(a) 
$$
\begin{align*}
\text{df}_E & = 25(16 - 1) = 375 \\
\text{MSE} & = \frac{19541}{375} \\
& = 52 \\
& = \hat{\sigma}_2^2 \\
\text{df}_A & = 25 - 1 = 24 \\
\text{MS}[A] & = \frac{11529}{24} \\
& = 480.4 \\
\hat{\sigma}_1^2 & = \frac{\text{MS}[A] - \text{MSE}}{16} \\
& = \frac{480.4 - 52}{16} \\
& = 26.8
\end{align*}
$$

(b) Use the normal approximation to the chisquare distribution ($\chi^2 n \approx N(n, 2n)$) to report an approximate 95% confidence interval for the variance component quantifying day-to-day variability (or variability not due to subject). The approximation $SSE/\sigma_2^2 \sim N(375, 750)$ leads to the 95% confidence interval

$$
\left( \frac{SSE}{428.6}, \frac{SSE}{321.3} \right) \text{ or } (45.6, 60.8)
$$

(c) Estimate the intraclass correlation coefficient and give two interpretations of this parameter estimate.

$$
\hat{\rho}_I = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2} = \frac{26.8}{26.8 + 52.1}
$$

(d) Estimate the mean diastolic blood pressure among males. Report a 95% confidence interval for this parameter. A 95% confidence interval given by

$$
\bar{y}_{++} \pm t(.025, 24) \sqrt{\frac{\text{MS}[A]}{400}} \text{ or } 91.7 \pm 2.26
$$
2. (30 pts) An experiment investigates the effect of row spacing on soybean yields. The design uses five row spacings (18, 24, 30, 36 or 42 inches apart) in a randomized complete block design with 6 plots. SAS code and output appropriate for this experiment appear at the end of this exam under the title “SOYBEAN PROBLEM”.

(a) Report the $F$-ratio and associated degrees of freedom for a test that row spacing has no effect on yield.

$$ F = \frac{MS[\text{spacing}]}{MSE} = \frac{126.5/4}{63.9/20} = 9.9 \text{ on } df = 4, 20 $$

(b) Report Tukey’s minimum significant difference (to control experimentwise error at $\alpha = .05$) for all pairwise comparisons among row spacing means.

$$ q(.05, 5, 20)\sqrt{\frac{MSE}{6}} = 4.23\sqrt{3.2/6} = 3.09 $$

(c) The coefficients for the linear polynomial contrast are $(-2, -1, 0, 1, 2)$. Obtain the $F$-ratio and associated degrees of freedom for a test of lack of fit in a model in which yield is linearly related to row spacing.

$$ SS(\theta) = \frac{\hat{\theta}^2}{10/6} = 96.8 $$

$$ SS(LOF) = SS(\text{spacing} - SS(\theta)) = (126.5 - 96.8) = 30.2 $$

$$ F_{LOF} = \frac{SS(LOF)/(4-1)}{MSE} = \frac{30.2}{10.1/3.2} = 3.14 \text{ on } df = 3, 20 $$
(30 pts) An experiment investigates the effects of two factors on corn yield. Three levels of nitrogen (56, 112, and 168 lbs/ha) and two types of irrigation (furrow, sprinkler) are used in a balanced 3 × 2 randomized design with a total of \( N = 24 \) plots. SAS code and output appropriate for this problem appears on a page entitled “CORNYIELDS PROBLEM” at the end of the exam.

(a) Report the \( F \)-ratio and \( p \)-value for a test that any effects of nitrogen on yield are constant across irrigation types.

\[
F_{irr\times nit} = 8.32, p = .0028
\]

(b) For each level of nitrogen, estimate the simple effect of irrigation type. Report standard errors and \( p \)-values for a test of no simple effects in each case.

<table>
<thead>
<tr>
<th>Nitrogen</th>
<th>irr effect</th>
<th>SE</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>4</td>
<td>4.14</td>
<td>0.35</td>
</tr>
<tr>
<td>112</td>
<td>26</td>
<td>4.14</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>168</td>
<td>7</td>
<td>4.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>

(c) For sprinkler irrigation, is the effect of nitrogen nonlinear? Report the quadratic (nonlinear) polynomial contrast for nitrogen (with coefficients \((1, -2, 1)\)) when sprinkler irrigation is used. Report the standard error.

\[
\hat{\theta}_{quad, sprinkler} = \bar{y}_{1+} - 2\bar{y}_{2+} + \bar{y}_{3+}
\]
\[
= -39
\]
\[
SE = \sqrt{6MSE/4}
\]
\[
= 7.2
\]
(d) Does the nonlinearity of the response to nitrogen vary across irrigation types? The quadratic polynomial contrast for furrow-irrigated corn is $\hat{\theta}_{\text{quad, furrow}} = 2$. Report the difference between the quadratic contrasts for furrow and sprinkler irrigated corn. Report the standard error for this difference.

$$\hat{\theta}_{\text{quad, sprinkler}} - \hat{\theta}_{\text{quad, furrow}} = 2 - (-39) = 41$$

$$SE = \sqrt{2MSE \times 6/4} = 10.1$$

(e) Draw a conclusion about the effects of nitrogen and irrigation type on yield, characterizing the nature of their interaction. Sketch a plot if it helps. The effect is linear for furrow-irrigated, but nonlinear for sprinkler irrigated. In particular, it looks like the sprinkler offers a big improvement with 112 lbs/ha of nitrogen.
SOYBEAN YIELDS PROBLEM

```latex
proc glm;
class block spacing;
model y=spacing block;
means spacing/tukey;
```

```
The SAS System
The GLM Procedure
Class Level Information

<table>
<thead>
<tr>
<th>Class</th>
<th>Levels</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>block</td>
<td>6</td>
<td>1 2 3 4 5 6</td>
</tr>
<tr>
<td>spacing</td>
<td>5 18 24 30 36 42</td>
<td></td>
</tr>
</tbody>
</table>

```

```
Sum of
Source | DF | Squares | Mean Square | F Value | Pr > F |
--------|----|---------|-------------|--------|--------|
Model   | ?  | 133.4333333 |             |        |        |
Error   | ?? | 71.5333333  |             |        |        |
Corrected Total | 29 | 204.9666667 |             |        |        |

```

```
R-Square | Coeff Var | Root MSE | y Mean |
-------- |-----------|----------|--------|
0.651000 | 6.029355  | 1.891208 | 31.36667 |

```

```
Source | DF | Type I SS | Mean Square | F Value | Pr > F |
--------|----|-----------|-------------|--------|--------|
spacing | ?  | 126.4666667 |             |        |        |
block    | ?  | 6.9666667  |             |        |        |

```

Tukey's Studentized Range (HSD) Test for y

```
NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha | 0.05
Error Degrees of Freedom | ??
Error Mean Square | ??
Critical Value of Studentized Range | 4.23186
Minimum Significant Difference | ?????

Means with the same letter are not significantly different.

```
Tukey Grouping | Mean | N | spacing |
----------------|------|---|---------|
A               | 35.167 | 6 | 18      |
B               | 31.833 | 6 | 24      |
B               | 30.333 | 6 | 30      |
B               | 30.000 | 6 | 42      |
B               | 29.500 | 6 | 36      |
```
proc glm order=dat;
class irrig nit;
model yield=irrig*nit;
lsmeans irrig*nit/slice=nit;
estimate "furrow: quad nit" nit 1 -2 1 irrig*nit 1 -2 1;

The SAS System
The GLM Procedure

Class Levels Values
irrig 2 furrow sprinkler
nit 3 56 112 168

Sum of
Source DF Squares Mean Square F Value Pr > F
Model 5 4539.333333 907.866667 26.53 <.0001
Error 18 616.000000 34.222222
Corrected Total 23 5155.333333

Source DF Type I SS Mean Square F Value Pr > F
irrig 1 912.666667 912.666667 26.67 <.0001
nit 2 3057.333333 1528.666667 44.67 <.0001
irrig*nit 2 569.333333 284.666667 8.32 0.0028

Least Squares Means

<table>
<thead>
<tr>
<th>irrig</th>
<th>nit</th>
<th>yield LSMEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>furrow</td>
<td>56</td>
<td>160.000000</td>
</tr>
<tr>
<td>furrow</td>
<td>112</td>
<td>171.000000</td>
</tr>
<tr>
<td>furrow</td>
<td>168</td>
<td>184.000000</td>
</tr>
<tr>
<td>sprinkler</td>
<td>56</td>
<td>164.000000</td>
</tr>
<tr>
<td>sprinkler</td>
<td>112</td>
<td>197.000000</td>
</tr>
<tr>
<td>sprinkler</td>
<td>168</td>
<td>191.000000</td>
</tr>
</tbody>
</table>

irrig*nit Effect Sliced by nit for yield

<table>
<thead>
<tr>
<th>nit</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>1</td>
<td>32.000000</td>
<td>32.000000</td>
<td>0.94</td>
<td>0.3464</td>
</tr>
<tr>
<td>112</td>
<td>1</td>
<td>1352.000000</td>
<td>1352.000000</td>
<td>39.51</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>168</td>
<td>1</td>
<td>98.000000</td>
<td>98.000000</td>
<td>2.86</td>
<td>0.1078</td>
</tr>
</tbody>
</table>

Standard

| Parameter | Estimate | Error | t Value | Pr > |t| |
|-----------|----------|-------|---------|------|-----|
| furrow: quad nit | 2.00000000 | 7.16472842 | 0.28 | 0.7833 |