Hypothesis Testing for $p$: Significance Levels, Errors, Power

1. A researcher developing scanners to search for hidden weapons at airports has concluded that a new scanner is not significantly better than the current scanner. He made his decision based on a test using $\alpha = .025$. Would he have made the same decision at $\alpha = .10$? How about $\alpha = .001$? Choose the correct answer below.
   a. His decision may have been different for $\alpha = .10$, but would have been the same for $\alpha = .001$.
   b. His decision may have been different for both $\alpha = .001$ and $\alpha = .10$.
   c. His decision may have been different for $\alpha = .001$, but would have been the same for $\alpha = .10$.
   d. His decision would have been the same for both $\alpha = .001$ and $\alpha = .10$.

2. A company is sued for job discrimination because only 17% of the newly hired candidates were women when 51% of all applicants were women.
   a. In this context, what would a Type I error be?
      A. A Type I error is deciding the company is discriminating when it is not.
      B. A Type I error is deciding the company is discriminating when, in fact, it is.
      C. A Type I error is deciding the company is not discriminating when, in fact, it is not.
      D. A Type I error is deciding the company is not discriminating when it is.
      E. There is no Type I error in this context.
   b. In this context, what would a Type II error be?
      A. A Type II error is deciding the company is discriminating when, in fact, it is.
      B. A Type II error is deciding the company is not discriminating when, in fact, it is not.
      C. A Type II error is deciding the company is not discriminating when it is.
      D. A Type II error is deciding the company is discriminating when it is not.
      E. There is no Type II error in this context.
c. If the hypothesis is tested at the 1% level of significance instead of 10%, how will this affect the power of the test?
A. The power of the test will decrease because the level of significance increased.
B. The power of the test will increase because the level of significance decreased.
C. The power of the test will decrease because the level of significance decreased.
D. The power of the test will increase because the level of significance increased.

d. The lawsuit is based on the hiring of 83 employees. Is the power of the test higher than, lower than, or the same as it would be if it were based on 57 hires?
A. The power of the test is higher because the sample size is greater.
B. The power of the test is lower because the sample size is greater.
C. The power of the test is the same regardless of the change in sample size.

3. Cellphone companies have discovered that college students, their biggest customers, have difficulty setting up all the features of their smart phones. Currently only 92% of students are able to set up all the features properly. To increase this percentage the cellphones companies have developed what they hope are new, simpler instructions. The new instructions are tested on 200 students, of whom 188 (94%) were successful. Is this evidence that the new instructions have increased the percentage of students who are able to set up all the features properly? To answer this question perform the hypothesis test $H_0 : p = .92 \quad H_A : p > .92$; use $\alpha = .05$.

a. Determine the $P$-value for this test. What is your conclusion?

$$ z = \frac{.94 - .92}{\sqrt{.92(1-.92)/200}} \approx 1.04. \quad P\text{-value} = P(z > 1.04) = .1492 $$

Do not reject $H_0$; there is no evidence to conclude that the proportion of students that are able to set up their smartphones is significantly larger than .92 when the new instructions are used.

b. What type of error might you be making? Type II

c. What is the power of the test when $p = .94$?

$$ P(z > 1.645 \text{ when } p = .94) = P\left(\frac{\hat{p} - .92}{\sqrt{.92(1-.92)/200}} > 1.645 \text{ when } p = .94\right) $$

$$ = P(\hat{p} > .952 \text{ when } p = .94) = P\left(\frac{\hat{p} - .94}{\sqrt{.94(1-.94)/200}} > \frac{.952 - .94}{\sqrt{.94(1-.94)/200}}\right) $$

$$ = P(z > .72) = .2358. $$

d. What is the power of the test when $p = .96$?

$$ P(z > 1.645 \text{ when } p = .96) = P\left(\frac{\hat{p} - .92}{\sqrt{.92(1-.92)/200}} > 1.645 \text{ when } p = .96\right) $$

$$ = P(\hat{p} > .952 \text{ when } p = .96) = P\left(\frac{\hat{p} - .96}{\sqrt{.96(1-.96)/200}} > \frac{.952 - .96}{\sqrt{.96(1-.96)/200}}\right) $$

$$ = P(z > -.58) = .7190. $$

e. What sample size is required if we want the power to be .80 when $p = .96$?
Rejection region in terms of \( \hat{p} \) for general sample size \( n \):
\[
z > 1.645 \Rightarrow \frac{\hat{p} - .92}{\sqrt{.92(1-.92)/n}} = \frac{\hat{p} - .92}{\frac{.92}{\sqrt{n}}} > 1.645 \Rightarrow \hat{p} > .92 + 1.645 \frac{.271}{\sqrt{n}}
\]
\[\Rightarrow \hat{p} > .92 + \frac{.446}{\sqrt{n}}.\]
\[.80 = P(\hat{p} > .92 + \frac{.446}{\sqrt{n}} \text{ when } p = .96)\]
\[= P\left(\frac{\hat{p} - .96}{\sqrt{.96(1-.96)/n}} > \frac{.92 + \frac{.446}{\sqrt{n}} - .96}{\sqrt{.96(1-.96)/n}}\right) = P\left(z > \frac{-0.04 + \frac{.446}{\sqrt{n}}}{\sqrt{.96(1-.96)/n}}\right)\]
\[= P(z > -0.204\sqrt{n} + \frac{.446}{1.96n}) = P(z > -0.204\sqrt{n} + 2.28)\]
\[\Rightarrow -0.842 = -0.204\sqrt{n} + 2.28 \Rightarrow \sqrt{n} = \frac{-0.842}{-0.204} = 15.3 \Rightarrow n = 234\]