ST705: Linear Models and Variance Components
Spring 2018

Web: http://www.stat.ncsu.edu/people/maity/courses/ST705/

Lecture: MoWe 11:45AM - 1:00PM  
Lab: Mo 10:40AM – 11:30AM  
Room: SAS Hall 1108

Instructor: Dr. Arnab Maity  
Office: SAS Hall 5240  
Office Hours: M 3:30 - 4:30pm

Teaching Assistant: Salil Koner  
Office Hours: TBA

Prerequisites: MA405 Linear Algebra; ST512R Statistical Methods; ST701 Probability Theory; ST702 Statistical Inference (co-requisite)

Textbook (required): Primer on Linear Models, J. Monahan, CRC Press

Additional References:
Theory and Application of the Linear Model, F. A. Graybill, Wadsworth  
Linear Models, S. R. Searle, Wiley  
Linear Regression Analysis, G. A. F. Seber, Wiley  
Matrix Algebra From A Statistician's Perspective, D. A. Harville, Springer  
Matrix Algebra as a Tool, A. Hadi, Duxbury  
The Matrix Cookbook, K. B. Petersen & M. S. Pedersen,  
http://www2.imm.dtu.dk/pubdb/views/publication_details.php?id=3274

Exams, Homework and Grading: There will be about 10 homework assignments, one midterm exam, and a final exam. The final grade will be computed from the scores with weighting: Homework, 30%; Midterm Exam, 30%; Final Exam, 40%.

Exams:
Midterm Exam: Monday February 26 (Lab + Lecture period).  
Final Exam: 5/4/2018, Friday; 8:00AM - 11:00AM; 01108 SAS Hall
Lecture Topics (The schedule of lecture topics is tentative and subject to change)

1. Basic Linear Algebra
   a. Vector and matrix: definition, operations and properties
   b. Linear combination of vectors
   c. Orthogonality, orthogonal matrix
   d. Vector space: definition, properties, dimension, span
   e. Null space and Column space
   f. Rank of a matrix, Inverse, trace, determinant
   g. Orthogonal complement
   h. Idempotent matrix, Projection matrix
   i. Matrix decompositions: QR, Cholesky, eigenvalues and eigenvectors, SVD

2. System of linear equations
   a. Solving
   b. Generalized inverse

3. General Linear model
   a. Definition
   b. Assumptions, different classes of models

4. Least Squares Method
   a. Optimization criterion
   b. Normal equations
   c. General solution
   d. Projection matrix onto C(X)

5. Reparameterization

6. Estimability
   a. Assumption of zero mean errors, preliminary discussion on identifiability
   b. Linear and unbiased estimators
   c. Linearly estimable functions
   d. Estimability in reparameterized models

7. Imposing constraints

8. The Gauss-Markov Model
   a. Random vectors, variance and covariance of random vectors and their linear transformations
   b. Model specification: mean zero, constant variance, uncorrelated errors
c. The Gauss-Markov theorem
d. Variance estimation
e. Model misspecification, underfitting and overfitting

9. The Aitken model
   a. Estimability
   b. Generalized least squares
   c. Aitken’s theorem
   d. Variance estimation
   e. Conditions under a linear estimator is BLUE

10. Best estimation in constrained model

11. Distributional theory
    a. Multivariate normal
    b. Chi square
    c. F and t
    d. Distribution of quadratic forms
    e. Cochran’s theorem

12. Hypothesis testing
    a. General linear hypothesis
    b. Testable hypothesis
    c. General linear test
    d. Likelihood ratio test

13. Confidence intervals and multiple comparisons
    a. Confidence intervals and regions
    b. Bonferroni method
    c. Scheffé’s method
    d. Tukey’s method
    e. Multiple testing

14. Identifiability

15. Linear mixed effects model
    a. ML and REML estimation
    b. ANOVA with random effects
    c. Best linear unbiased prediction