PAIRED COMPARISON DESIGNS FOR TESTING CONCORDANCE BETWEEN JUDGES

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This research was jointly supported by the United States Air Force through the Office of Scientific Research of the Air Research and Development Command, and the Division of Research Techniques, London School of Economics and Political Science.

Institute of Statistics
Mimeograph Series No. 134
June 22, 1955

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1. Summary and Introduction.

In a recent paper, "Further contributions to the theory of paired comparisons", M. G. Kendall (1955), considers paired comparison designs, in which each pair of judges have certain comparisons in common. Such designs should prove useful for testing concordance between judges. He notes that designs of an optimum kind which balance by numbers of comparisons, objects compared, numbers of observers on given comparisons and so forth are rather rare. It is the object of this paper to obtain some paired comparison designs which have a high degree of symmetry. These designs have been defined in Section 2, and certain inequalities between the parameters are obtained in Section 3. In Sections 4 and 5, two special classes of these designs have been investigated, and explicit designs for small values of n (the number of objects to be compared) have been given in Tables I and II. The method of analysis would, to a certain extent, depend on what use the experimenter wants to make of the design. This question will be considered in a future communication.

My thanks are due to Professor Kendall for suggesting the problem, and for helpful discussion during the preparation of the paper.

2. Definition of linked paired comparison designs.

Suppose it is required to compare n objects, by employing v judges. Each judge compares r pairs of objects ($r > 1$), and in respect of each pair expresses his opinion whether he prefers one or the other object of the pair. In certain circumstances it may be desirable to allow the judge to express no preference with respect

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to either of the objects forming the pair. In this case we say that the preference is equally shared by the two objects. We shall assume that the pairs compared by any judge are all different. To ensure symmetry between objects and judges, we require that :

(a) Among the r pairs, compared by each judge, each object appears equally often, say α times.

(b) Each pair is compared by k judges, $k > 1$.

(c) Given any two judges there are exactly λ pairs which are compared by both judges.

Designs satisfying these conditions may be called <u>linked paired comparison designs</u>.

Clearly

$$r = \frac{n\alpha}{2} \quad . \tag{2.1}$$

The number of possible pairs is

$$b = n(n - 1)/2. \tag{2.2}$$

There is a certain correspondance between linked paired comparison designs and balanced incomplete block designs. Each judge may be considered to correspond to a treatment, and each pair to a block. If a pair is compared by a judge, then the block corresponding to the pair may be considered to contain the treatment corresponding to the judge. Hence if a linked paired comparison design of the type considered exists there must exist a corresponding balanced incomplete block design with v treatments and $b = n(n - 1)/2$ blocks, such that each block contains k treatments, each treatment occurs in r blocks, and two given treatments occur together in λ blocks. It follows at once that

$$bk = vr, \quad \lambda(v - 1) = r(k - 1). \tag{2.3}$$

Using (2.1) and (2.2), the first of the relations (2.3) can be written as

$$v\alpha = k(n - 1). \tag{2.4}$$

Also Fisher's well-known inequality [Fisher (1940), Bose (1949)] gives

(2.5) $$b \geq v \text{ or } r \geq k$$

(2.6) $$\therefore \qquad n\alpha \geq 2k .$$

It should be remembered that the existence of a balanced incomplete block design with parameters v, b = n(n - 1)/2, r, k, λ does not ensure the existence of a corresponding linked paired comparison design due to the additional restriction (a). Clearly

(2.7) $$r \geq \lambda .$$

The case r = λ is trivial, since in this case all the r pairs compared by any judge, must also be compared by every other judge. This means that each judge compares the same pairs. Condition (b) then shows that there must be exactly k judges, and each judge compares every pair so that r = n(n - 1)/2.

3. Some inequalities.

From (2.1) and (2.4)

$$v = \frac{k}{\alpha}\left(\frac{2r}{\alpha} - 1\right) .$$

Hence for (2.3)

(3.1) $$\lambda = \frac{\alpha^2 r(k-1)}{k(2r-\alpha)-\alpha^2} = \frac{\alpha^2}{2} + \frac{\alpha^2(k\alpha+\alpha^2-2r)}{2(2kr-k\alpha-\alpha^2)} .$$

This shows that for a given k, λ decreases as r increases. Now r ≥ λ, and when r = λ, we get from (3.1),

$$r = \lambda = \frac{\alpha(\alpha+1)}{2} .$$

This proves that

(3.2) $$r \geq \frac{\alpha(\alpha+1)}{2} , \qquad \lambda \leq \frac{\alpha(\alpha+1)}{2}$$

where the equality holds only in the trivial case r = λ.

Since λ must be a positive integer, (3.2) shows that α = 1 must imply r = λ = α = 1. Each judge therefore compares only one pair. Hence the case α = 1 is impossible except in the trivial case when there are only two objects and each judge compares them.

Again we can write (3.1) as

$$\lambda = \frac{(k-1)\alpha^2}{2k} + \frac{\alpha^3(k-1)(k+\alpha)}{2k(2kr-k\alpha-\alpha^2)} \tag{3.3}$$

It follows from (3.2) that

$$2kr - k\alpha - \alpha^2 \geq (k-1)\alpha^2$$

Hence the second term in (3.3) is positive. We thus have the inequality

$$\lambda > \frac{(k-1)\alpha^2}{2k} \tag{3.4}$$

Since $k \geq 2$, combining (3.4) and (3.1), we have

$$\frac{\alpha^2}{4} < \lambda \leq \frac{\alpha(\alpha+1)}{2} \tag{3.5}$$

$$\frac{d\lambda}{dk} = \frac{\alpha^2 r(2r-\alpha-\alpha^2)}{(2kr-k\alpha-\alpha^2)^2} \tag{3.6}$$

It follows from (3.2) that

$$\frac{d\lambda}{dk} \geq 0 . \tag{3.7}$$

Hence λ is a monotonically increasing function of k, for a given r.

4. Linked paired comparison designs with $\alpha = 2$.

The inequality (3.5) shows that when $\alpha = 2$ (neglecting the trivial case $r = \lambda$), we must have $\lambda = 2$. Using (2.1), (2.2), (2.3) and (2.4), we see that all the parameters of the design can be expressed in terms of n, the number of objects to be compared. In fact we have

$$v = \frac{(n-1)(n-2)}{2},\ b = \frac{n(n-1)}{2},\ r = n,\ k = n-2,\ \lambda = 2,\ \alpha = 2 . \tag{4.1}$$

The existence of (4.1), implies the existence of the balanced incomplete design with parameters

$$v = \frac{(n-1)(n-2)}{2},\ b = \frac{n(n-1)}{2},\ r = n,\ k = n-2,\ \lambda = 2. \tag{4.2}$$

The balanced incomplete block design (4.2) is known to exist for the values $n = 4, 5, 6$ and 9 [Fisher and Yates (1938), Bose (1939)], and can in fact be derived by first writing down a solution of the symmetrical balanced incomplete block design,

$$(4.3) \qquad v = b = \frac{n(n-1)}{2} + 1, \qquad r = k = n, \qquad \lambda = 2$$

and then deleting one block and all the treatments in this block. This results in two treatments being deleted from each of the other $n(n-1)/2$ blocks, since any block of (4.3) has exactly $\lambda = 2$ treatments common with every other block. Conversely, it is known that the existence of (4.2) implies the existence of (4.3) [Connor(1952), Connor and Hall (1954)]. Hence the non-existence of (4.3) for any value of n, implies the non-existence of (4.2) for the same values of n. Certain sufficient conditions for the non-existence of (4.3) are known [Shrikhande (1950, Chowla and Rysér (1950)]. In particular the cases n = 7, 8 and 10 are impossible. The design (4.2) is called the derived of (4.3).

We shall now show that for any solution of the symmetrical balanced incomplete block design (4.3), we can derive a corresponding solution of the linked paired comparison design (4.1). The process of derivation will first be illustrated by considering the special case n = 6.

It is known [Carmichael (1937)], that a solution for (4.3)in the special case n = 6, can be obtained by writing down sixteen treatments in the cells of a 4 x 4 square and then taking for blocks the six treatments which occur in the same row or the same column as a given treatment (but not including in the block this treatment itself). We shall take the sixteen treatments to be 0, 1, 2, 3, 4, 5, 6 and a, b, c, d, e, f, g, h, k and arrange them as shown below

(4.4)

0	1	2	3
4	a	b	c
5	d	e	f
6	g	h	k

Then the 16 blocks of (4.3) for the case n = 6, are given by

(4.5)

1,	2,	3,	4,	5,	6
1,	2,	0,	c,	f,	k
1,	3,	0,	b,	e,	h
1,	4,	b,	c,	d,	g
1,	5,	a,	g,	e,	f
1,	6,	a,	d,	h,	k
2,	3,	0,	a,	d,	g
2,	4,	a,	c,	e,	h
2,	5,	b,	h,	d,	f
2,	6,	e,	b,	g,	k
3,	4,	a,	b,	f,	k
3,	5,	c,	k,	d,	e
3,	6,	c,	f,	g,	h
4,	5,	0,	g,	h,	k
4,	6,	0,	d,	e,	f
5,	6,	0,	a,	b,	c

A solution of the derived design (4.2) for the case n = 6 is now obtainable from (4.5) by omitting the first block in (4.5) and the treatments 1, 2, 3, 4, 5, 6 from the remaining blocks. Thus the derived design is the part included within the thick lines in (4.5). We can make a (1,1) correspondence between the blocks of the derived design and the unordered pairs (i,j), i,j = 1, 2, 3, 4, 5, 6; $i \neq j$, since each of these blocks has been obtained by deleting just one such pair, from the corresponding block of (4.3). Thus the block 0, c, f, k corresponds to the pair (1,2) and so on. Now let us identify the ten treatments 0, a, b, c, d, e, f, g, h and k of the derived with 10 judges, and assign to each judge, the pairs corresponding to the blocks, in which the treatment corresponding to the judge occurs. We thus get the linked paired comparison design No. (3) of Table I. Since each of the treatments 0, a, b, c, d, e, f, g, h, k occurs in exactly $r = 6$ blocks of the derived, each judge is assigned exactly six pairs, and since any pair of treatments occurs in exactly $\lambda = 2$ blocks, any two judges have just two pairs in common. Again, since each pair of the derived has exactly $k = 4$ treatments, each pair is compared by just 4 judges. Finally, let i be one of the six deleted treatments 1, 2, 3, 4, 5, 6 and let x stand for one of the ten treatments of the derived. Then i and x occur together in only two blocks of the symmetrical design, for example,1 and a occur together only in the blocks (1, 5, a, g, e, f) and (1, 6, a, d, h, k). Hence the object i occurs twice among the pairs compared by the judge x. This shows that each of the six objects 1, 2, 3, 4, 5, 6 occurs twice among the pairs compared by a judge.

The same process can be used for obtaining a solution of (4.1) for any value of n, for which a solution of the symmetrical balanced incomplete block design (4.3) is known. We first rename the treatments in such a way that the first block contains the treatments 1, 2,..., n. Then each of the remaining $n(n-1)/2$ blocks contains just one of the unordered pairs (i,j), $i,j = 1,2,\ldots,n$, $i \neq j$. The remaining $(n-1)(n-2)/2$ treatments may then be identified with judges. If x be one of these treatments, then the judge x is assigned pairs (i,j) corresponding to those blocks in which x occurs.

Cyclic solutions to the symmetrical balanced incomplete block designs (4.3) are known for the cases $n = 4$, 5 and 9. All the blocks can be obtained by developing a suitable initial block mod v. These initial blocks are given below

n	Initial block	
4	(0, 3, 5, 6)	mod 7
5	(1, 4, 5, 9, 3)	mod 11
9	(1, 16, 34, 26, 9, 33, 10, 12, 7)	mod 37

The corresponding linked paired comparison designs are the designs No. (1), (2) and (4) of Table I.

TABLE I

No.	Parameters	Judge	Design: Pairs assigned to a judge
(1)	$n = 4, \quad v = 3$	a	(1,4), (1,3), (2,4), (2,3)
	$b = 6, \quad r = 4$	b	(1,3), (2,4), (1,2), (3,4)
	$k = 2, \quad \lambda = 2$	c	(1,4), (1,2), (2,3), (3,4)
	$\alpha = 2$		
(2)	$n = 5, \quad v = 6$	a	(3,5), (2,4), (1,3), (1,4), (2,5)
	$b = 10, r = 5$	b	(2,3), (3,4), (1,4), (1,5), (2,5)
	$k = 3, \quad \lambda = 2$	c	(2,3), (3,5), (1,2), (4,5), (1,4)
	$\alpha = 2$	d	(3,5), (1,2), (3,4), (2,4), (1,5)
		e	(1,2), (3,4), (4,5), (1,3), (2,5)
		f	(2,3), (4,5), (2,4), (1,3), (1,5)
(3)	$n = 6, \quad v = 10$	0	(1,2), (1,3), (2,3), (4,5), (4,6), (5,6)
	$b = 15, r = 6$	a	(2,3), (2,4), (3,4), (1,5), (1,6), (5,6)
	$k = 4, \quad \lambda = 2$	b	(1,3), (1,4), (3,4), (2,5), (2,6), (5,6)
	$\alpha = 2$	c	(1,4), (2,4), (1,2), (3,5), (3,6), (5,6)
		d	(1,4), (1,6), (4,6), (2,3), (2,5), (3,5)
		e	(1,3), (1,5), (3,5), (2,4), (2,6), (4,6)
		f	(1,2), (1,5), (2,5), (3,4), (3,6), (4,6)
		g	(1,4), (1,5), (4,5), (2,3), (2,6), (3,6)
		h	(1,3), (1,6), (3,6), (2,4), (2,5), (4,5)
		k	(1,2), (1,6), (2,6), (3,4), (3,5), (4,5)

Table I (continued)

Linked paired designs with $\alpha = 2$			
No.	Parameters	Design: Judge	Pairs
(4)	n=9, v=28	a	(5,6), (6,7), (7,8), (8,9), (1,9), (1,2), (2,3), (3.4), (4,5)
	b=36,r=9	b	(3,7), (6,8), (1,8), (5,7), (4,5), (2,9), (1,4), (2,3), (6,9)
	k=7, λ=2	c	(5,8), (3,6), (2,9), (4,7), (1,7), (2,6), (4,5), (1,9), (3,8)
	α=2	d	(7,8), (3,4), (2,6), (2,8), (6,9), (3,5), (1,7), (1,4), (5,9)
		e	(1,2), (1,6), (3,5), (4,8), (3,8), (2,7), (6,9), (4,5), (7,9)
		f	(1,8), (2,3), (2,7), (4,6), (5,9), (4,9), (3,8), (1,7), (5,6)
		g	(2,6), (1,4), (8,9), (2,4), (5,6), (1,5), (7,9), (3,8), (3,7)
		h	(4,9), (6,9), (4,7), (1,3), (5,8), (2,8), (3,7), (5,6), (1,2)
		i	(1,5), (5,9), (4,8), (3,6), (1,2), (4,6), (7,8), (3,7), (2,9)
		j	(5,7), (7,9), (4,6), (3,4), (1,8), (3,9), (1,2), (5,8), (2,6)
		k	(4,7), (5,6), (3,9), (1,6), (2,9), (2,4), (1,8), (7,8), (3,5)
		l	(4,8), (3,7), (2,5), (1,9), (3,5), (6,7), (2,6), (1,8), (4,9)
		m	(4,6), (5,8), (6,7), (1,4), (2,7), (1,3), (3,5), (2,9), (8,9)
		n	(3,9), (7,8), (1,3), (4,5), (4,9), (6,8), (2,7), (2,6), (1,5)
		o	(2,4), (1,2), (6,8), (1,7), (8.9), (3,6), (4,9), (3,5), (5,7)
		p	(2,5), (1,8), (3,6), (6,9), (1,5), (3,4), (8,9), (2,7), (4,7)
		q	(6,7), (2,9), (3,4), (3,8), (5,7), (1,6), (1,5), (4,9), (2,8)
		r	(1,3), (2,6), (1,6), (5,9), (4,7), (2,3), (5,7), (8,9), (4,8)
		s	(6,8), (3,5), (2,3), (7,9), (2,8), (1,9), (4,7), (1,5), (4,6)
		t	(3,6), (2,7), (1,9), (5,6), (4,8), (1,4), (2,8), (5,7), (3,9)
		u	(1,6), (8,9), (4,5), (3,7), (3,9), (1,7), (4,6), (2,8), (2,5)
		v	(2,3), (1,5), (1,7), (5,8), (2,4), (6,9), (3,9), (4,8), (6,7)
		w	(1,9), (5,7), (6,9), (7,8), (2,5), (3,8), (2,4), (4,6), (1,3)
		x	(1,4), (4,7), (3,8), (1,2), (6,7), (5,9), (2,5), (3,9), (6,8)
		y	(4,5), (2,8), (5,9), (1,8), (1,3), (7,9), (6,7), (2,4), (3,6)
		z	(1,7), (4,8), (7,9), (2,9), (6,8), (5,6), (1,3), (2,5), (3,4)
		α	(5,9), (2,4), (3,7), (2,7), (1,6), (5,8), (3,4), (6,8), (1,9)
		β	(7,9), (2,5), (5,8), (4,9), (2,3), (7,8), (1,6), (3,6), (1,4)

5. Some other types of linked paired designs.

Let the number of objects n be even, say n = 2t. Then we can divide the t(2t-1) pairs, into 2t-1 sets of t pairs each, such that each object occurs exactly once among the pairs of a set. For example, if n = 8, we can take the objects to be 0, 1, 2, 3, 4, 5, 6 and ∞. Then the seven sets are

	Sets	Pairs
	I	$(1,6),(2,5),(3,4),(0,\infty)$
	II	$(2,0),(3,6),(4,5),(1,\infty)$
	III	$(3,1),(4,0),(5,6),(2,\infty)$
(5.1)	IV	$(4,2),(5,1),(6,0),(3,\infty)$
	V	$(5,3),(6,2),(0,1),(4,\infty)$
	VI	$(6,4),(0,3),(1,2),(5,\infty)$
	VII	$(0,5),(1,4),(2,3),(6,\infty)$

In the general case, the 2t-1 sets can be obtained by developing mod(2t-1), the initial set

(5.2) $$(1,2t-2),\ (2,25-3),\ldots,(t-1,t),\ (0,\infty)$$

the object ∞ remaining unchanged.

Let us now take a balanced incomplete block design with v^* treatments, $b^* = 2t-1$ blocks, r^* replications, block size k^* and in which every pair of treatments occurs together in the same block λ^* times, and make each block correspond to one set and each treatment correspond to one judge. We can then obtain a linked paired comparison design by assigning to each judge, the sets of pairs corresponding to all blocks in which the treatment corresponding to the judge occurs. We obtain in this way a linked paired comparison design with parameters

(5.3) $$n = 2t,\ v = v^*,\ b = t(2t-1),\ r = tr^*,\ k = k^*,\ \lambda = t\lambda^*,\ \alpha = r^* .$$

For example, in the case n = 8, we may start with the balanced incomplete block design with parameters

$$v^* = 7,\ b^* = 7,\ r^* = 3,\ k^* = 3,\ \lambda^* = 1 .$$

If the treatments are taken as a, b, c, d, e, f, g, then the blocks are

(5.4)

a,	b,	d
b,	c,	e
c,	d,	f
d,	e,	g
e,	f,	a
f,	g,	b
g,	a,	c

If we make them correspond to the sets I - VII given by (5.1), then we get the following linked paired comparison design

(5.5)

Judge	Sets of Pairs
a	I, II, IV
b	II, III, V
c	III, IV, VI
d	IV, V, VII
e	V, VI, I
f	VI, VII, II
g	VII, I, III

The parameters of the design are

$$n = 8,\ v = 7,\ b = 28,\ r = 12,\ k = 4,\ \lambda = 4,\ \alpha = 3.$$

Each judge compares 12 of the 28 possible pairs. Other designs obtained in this way are the designs No. (1), (2) and (4) of Table II. It should be noticed that the design (1) of Table II is the same as the design (1) of Table I, obtained in a different manner.

Again let the number of objects be odd, say $n = 2t+1$. Then we can divide the $t(2t+1)$ pairs into t sets of $(2t+1)$ pairs each, such that each object occurs exactly twice among the pairs of a set. For example, if $n = 7$, we can take the objects to be 0, 1, 2, 3, 4, 5, 6. Then the three sets are:

	Sets	Pairs
	I	(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,0)
(5.6)	II	(0,2),(1,3),(2,4),(3,5),(4,6),(5,0),(6,1)
	III	(0,3),(1,4),(2,5),(3,6),(4,0),(5,1),(6,2) .

In the general case we can take the objects to be 0, 1,...,2t. Then the i-th set consists of all pairs for which the difference mod(2t+1) between the objects constituting the pair is i.

Let us now take a balanced incomplete block design with v' treatments, b' = t blocks, r' replications, block size k' and in which every pair of treatments occurs together in the same block λ' times, and make eash block correspond to one set and each treatment to a judge. We then get a linked paired comparison design by assigning to each judge, the sets of pairs corresponding to all blocks, in which the treatment corresponding to the judge occurs. We obtain in this way a linked paired design with parameters

(5.7) $\quad n = 2t+1,\ v = v',\ b = t(2t+1),\ r = (2t+1)r',\ k = k',\ \lambda = (2t+1)\lambda',\alpha = 2r'$.

For example, in the case n = 7, we get the linked paired comparison design No. (5) of Table II. It should be noted that the sets obtained in this case are the same as the tours round the preference polygon considered by Kendall (1955), and lead to the designs already considered by him.

Table II

No.	Parameters	Sets of pairs	Design: Judge	Design: Sets of pairs assigned to a judge
(1)	$n=4,\ v=3$	I $(1,2),\ (0,\infty)$	a	II, III
	$b=6,\ r=4$	II $(2,0),\ (1,\infty)$	b	III, I
	$k=2,\ \lambda=2$	III $(0,1),\ (2,\infty)$	c	I, II
	$\alpha=2$			
(2)	$n=6,\ v=5$	I $(1,4),\ (2,3),\ (0,\infty)$	a	II, III, IV, V
	$b=15,\ r=12$	II $(2,0),\ (3,4),\ (1,\infty)$	b	I, III, IV, V
	$k=4,\ \lambda=9$	III $(3,1),\ (4,0),\ (2,\infty)$	c	I, II, IV, V
	$\alpha=4$	IV $(4,2),\ (0,1),\ (3,\infty)$	d	I, II, III, V
		V $(0,3),\ (1,2),\ (4,\infty)$	e	I, II, III, IV
(3)	$n=8,\ v=7$			
	$b=28,\ r=12$			
	$k=4,\ \lambda=4$	The sets (5.1)		The design (5.5)
	$\alpha=3$			
(4)	$n=8,\ v=7$		a	III, V, VI, VII
	$b=28,\ r=16$		b	IV, VI, VII, I
	$k=4,\ \lambda=8$	The sets (5.1)	c	V, VII, I, II
	$\alpha=4$		d	VI, I, II, III
			e	VII, II, III, IV
			f	I, III, IV, V
			g	II, IV, V, VI
(5)	$n=27,\ v=3$		a	II, III
	$b=21,\ r=14$	The sets (5.6)	b	III, I
	$k=2,\ \lambda=7$		c	I, II
	$\alpha=4$			

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