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INFORMATION MATRIX FOR A MIXTURE
OF TWO NORMAL DISTRIBUTIONS

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ABSTRACT

This paper presents a numerical method for computation of the Fisher information matrix about the five parameters $\mu_1, \mu_2, p, \sigma_1^2, \sigma_2^2$ of a mixture of two normal distributions. It is shown, by using a simple transformation which reduces the number of parameters from five to three, that the computation of the whole information matrix leads to the numerical evaluation of a particular integral. The Hermite-Gauss quadrature formula, the Romberg's algorithm, a power series, and Taylor's expansion are applied for the evaluation of this integral and the results are compared with each other.

A short table has been provided from which the approximate information matrix can be obtained in practice.

1. INTRODUCTION

Consider

$$f(x) = pf_1(x) + qf_2(x) \quad (1.1)$$

where $f_j(x)$ is a normal density with parameters μ_j and σ_j^2 , $j = 1, 2$, with $0 < p < 1$ and $q = 1-p$. The function $f(x)$ is the probability density function of a mixture of two normal distributions with mixing proportions p and q . The point estimation of the parameters of this distribution has recently received some attention in the literature [1, 3, 4, 6]. However, due to

the complexity of the problem, there is no simple way to measure the precision of the estimators obtained by different methods or to consider optimal sample size for specified precision. A convenient statistical measure which can be used for assessing the sample size before experiment and evaluating the approximate precision after estimation is the Fisher information, whose properties are well-known.

This paper presents a numerical method for computation of the Fisher information matrix about the five parameters $\mu_1, \mu_2, p, \sigma_1^2, \sigma_2^2$ of a mixture of two normal distributions, which can even be extended to finite mixtures of more than two normal distributions. The first work related to this paper is by Hill [8], who gives a general power series for computing the Fisher information of the proportion p in a mixture of two densities. In particular, he considers the case of exponential mixture and normal mixture with equal variances in more detail. The information matrix is also computed by the author in [2] for the special case of a normal mixture with equal variances.

Our attempt here is to show that the computation of the whole information matrix leads to the numerical evaluation of a particular integral, by using a simple transformation which reduces the number of parameters from five to three positive ones. The Hermite-Gauss quadrature formula, the Romberg's algorithm, a power series, and Taylor's expansion are applied for the evaluation of this integral and the results are compared with each other. A short table has been provided from which the information matrix can be easily computed in practice.

2. INFORMATION MATRIX

Let X be a random variable with the probability density function $f(x, \theta)$ depending on an h -dimensional parameter $\theta = (\theta_1, \theta_2, \dots, \theta_h)$. The information matrix that we shall use is that of Fisher, namely, the symmetric positive definite $h \times h$ matrix $I(\theta) = ||I(\theta_s, \theta_t)||$ with

$$I(\theta_s, \theta_t) = E \left[\frac{\partial \log f(x, \theta)}{\partial \theta_s} \cdot \frac{\partial \log f(x, \theta)}{\partial \theta_t} \right], \quad s, t = 1, 2, \dots, h, \quad (2.1)$$

where the expectation is taken with respect to the density $f(x, \theta)$, and it is assumed that all the derivatives and expectations in question exist.

Simple calculation shows that for the density (1.1) the following relations hold for $i = 1, 2$.

$$\begin{aligned} \frac{\partial \log f(x)}{\partial \mu_i} &= \frac{p_i}{\sigma_i} \left(\frac{x - \mu_i}{\sigma_i} \right) \frac{f_i(x)}{f(x)} \\ p \frac{\partial \log f(x)}{\partial p} &= 1 - \frac{f_2(x)}{f(x)} \\ \frac{\partial \log f(x)}{\partial \sigma_i^2} &= \frac{p_i}{2\sigma_i^2} \left[\left(\frac{x - \mu_i}{\sigma_i} \right)^2 - 1 \right] \frac{f_i(x)}{f(x)}, \end{aligned} \quad (2.2)$$

where $p_1 = p$ and $p_2 = q$. For $i, j = 1, 2$ and $m, n = 0, 1, 2$, let

$$M_{mn}(f_i, f_j) = \int_{-\infty}^{\infty} \left(\frac{x - \mu_i}{\sigma_i} \right)^m \left(\frac{x - \mu_j}{\sigma_j} \right)^n \frac{f_i(x) f_j(x)}{f(x)} dx. \quad (2.3)$$

It is easy to show that the improper integrals (2.3) exist. Now, using (2.1)-(2.3), we obtain the following items, which we call the scaled elements of the information matrix.

$$\sigma_i \sigma_j I(\mu_i, \mu_j) = p_i p_j M_{11}(f_i, f_j)$$

$$\sigma_i I(\mu_i, p) = -M_{10}(f_i, f_2)$$

$$\begin{aligned}
\sigma_2 I(\mu_2, p) &= M_{01}(f_1, f_2) \\
\sigma_1 \sigma_1^2 I(\mu_1, \sigma_1^2) &= \frac{pp_1}{2} [M_{12}(f_1, f_1) - M_{10}(f_1, f_1)] \\
\sigma_1^2 \sigma_2 I(\mu_2, \sigma_1^2) &= \frac{p_1 q}{2} [M_{21}(f_1, f_2) - M_{01}(f_1, f_2)] \\
I(p, p) &= \frac{1}{pq} [1 - M_{00}(f_1, f_2)] \\
\sigma_1^2 I(p, \sigma_1^2) &= \frac{1}{2} [M_{00}(f_1, f_2) - M_{20}(f_1, f_2)] \\
\sigma_2^2 I(p, \sigma_2^2) &= \frac{1}{2} [M_{02}(f_1, f_2) - M_{00}(f_1, f_2)] \\
\sigma_1^2 \sigma_2^2 I(\sigma_1^2, \sigma_2^2) &= \frac{pq}{4} [M_{00}(f_1, f_2) - M_{20}(f_1, f_2) - M_{02}(f_1, f_2) + M_{22}(f_1, f_2)] \\
\sigma_1^4 I(\sigma_1^2, \sigma_1^2) &= \frac{p_1^2}{4} [M_{00}(f_1, f_1) - 2M_{11}(f_1, f_1) + M_{22}(f_1, f_1)].
\end{aligned} \tag{2.4}$$

Denoting the information matrix, in the order $\mu_1, \mu_2, p, \sigma_1^2, \sigma_2^2$ for the parameters, by $I = ||I(s, t)||$ and the scaled information matrix by $J = ||J(s, t)||$, $s, t = 1, \dots, 5$, and considering the diagonal matrix $\Delta = \text{diag.}(1/\sigma_1, 1/\sigma_2, 1, 1/\sigma_1^2, 1/\sigma_2^2)$, we have

$$I = \Delta J \Delta. \tag{2.5}$$

Hence to compute I , it is enough to compute J and then apply (2.5).

3. A TRANSFORMATION FOR COMPUTING THE MATRIX J

To compute the matrix J , we have to find the numerical values of the integrals (2.3) which depend on five parameters $\mu_1, \mu_2, p, \sigma_1^2, \sigma_2^2$. However, the following simple transformation shows that these integrals, in fact, are functions of only three positive parameters up to a sign factor.

Let $\sigma_1 \leq \sigma_2$ without loss of generality. Now consider the linear transformation

$$y = \varepsilon(x - \bar{\mu}) / \bar{\sigma} \tag{3.1}$$

assuming that $\epsilon = 1$ for $\mu_1 \leq \mu_2$ and $\epsilon = -1$ for $\mu_1 > \mu_2$, where

$$\bar{\mu} = (\mu_1 + \mu_2)/2, \quad \bar{\sigma} = \sqrt{\sigma_1 \sigma_2}. \quad (3.2)$$

Take also

$$D = |\mu_2 - \mu_1|/2\bar{\sigma}, \quad r = \sigma_1/\sigma_2, \quad (3.3)$$

where $D \geq 0$ and $0 < r \leq 1$. One can easily show that the transformation (3.1) sends the density function (1.1) to the density function

$$g(y) = pg_1(y) + qg_2(y), \quad (3.4)$$

where, for $i = 1, 2$,

$$g_i(y) = (1/\sqrt{2\pi r_i}) \exp[-(y-D_i)^2/2r_i] \quad (3.5)$$

with the conventions $D_1 = -D$, $D_2 = D$, $r_1 = r$, and $r_2 = 1/r$. Thus, by using the transformation (3.1), the normal mixture (1.1) with five parameters is reduced to (3.4), say, the standard normal mixture with three positive parameters D , p , r .

It is obvious that under the linear transformation (3.1) the formula (2.3) becomes

$$M_{mn}(f_i, f_j) = \epsilon^{m+n} r_i^{-m/2} r_j^{-n/2} G_{mn}(g_i, g_j) \quad (3.6)$$

with

$$G_{mn}(g_i, g_j) = \int_{-\infty}^{\infty} (y-D_i)^m (y-D_j)^n g_i(y) g_j(y)/g(y) dy. \quad (3.7)$$

As the formulae (2.4) show, the sign factor ϵ^{m+n} only affects the elements $I(\mu_i, p)$ and $I(\mu_i, \sigma_j^2)$, for $i, j = 1, 2$, when $\mu_1 > \mu_2$. Thus, to make the tabulation of the results easier, we can assume, as well, that $\mu_1 \leq \mu_2$ and

eliminate ε^{m+n} ; whenever $\mu_1 > \mu_2$, we just give a minus sign to the above mentioned elements which depend on μ_1 and μ_2 through $D = |\mu_2 - \mu_1|/2\bar{\sigma}$.

4. HERMITE-GAUSS QUADRATURE AND ROMBERG'S ALGORITHM FOR COMPUTATION OF $G_{mm}(g_1, g_j)$

The Hermite-Gauss quadrature formula for a convergent integral of the form

$$I(G) = \int_{-\infty}^{\infty} e^{-z^2} G(z) dz \quad (4.1)$$

is

$$I(G) = \sum_{k=1}^N w_k G(z_k) + R_N(G), \quad (4.2)$$

where z_k and w_k , which are referred to respectively as nodes and weights, can be determined through some appropriate Hermite polynomials and they are already known and tabulated [7, 9]. The summation $\sum_{k=1}^N w_k G(z_k)$ approximates $I(G)$ with the remainder

$$R_N(G) = N! \sqrt{\pi} G^{(2N)}(\bar{z})/2^N (2N)!, \quad (4.3)$$

where \bar{z} is a real number. In general, the size of $R_N(G)$ depends on the behavior of the derivatives of $G(z)$ over $(-\infty, \infty)$, and it cannot be easily estimated. To guarantee a sufficiently small error, it is desirable that these derivatives remain bounded with a small supremum over $(-\infty, \infty)$.

Now we show that the integral (3.7), by introducing the suitable factor $\exp(-ry^2/4)$, can be put in the form (4.1) with a bounded function $G(z)$ and bounded derivatives $G^{(N)}(z)$. For this purpose we write

$$g_1(y)g_j(y)/g(y) = \exp(-ry^2/4) / [pg_1(y)g_1^{-1}(y)g_j^{-1}(y) + qg_2(y)g_1^{-1}(y)g_j^{-1}(y)] \cdot \exp(-ry^2/4), \quad (4.4)$$

and we use the transformation $z = y\sqrt{r}/2$ to obtain

$$G_{mn}(g_i, g_j) = c_{mnij} \int_{-\infty}^{\infty} e^{-z^2} G_{mnij}(z) dz, \quad (4.5)$$

where simple calculation shows that for $m, n = 0, 1, 2$ and $i, j = 1, 2$,

$$\begin{aligned} c_{mnij} &= r^{-(m+n)/2} r_i^{-1/2} r_j^{-1/2} \sqrt{2/\pi} \\ G_{mnij}(z) &= (2z - D_i \sqrt{r})^m (2z - D_j \sqrt{r})^n / [p \exp Q_{ij1}(z) + q \exp Q_{ij2}(z)] \\ Q_{ijk}(z) &= A_{ijk} z^2 + B_{ijk} z + C_{ijk} \\ A_{ijk} &= 2(1/r_i + 1/r_j - 1/r_k - r/2)/r \\ B_{ijk} &= -2(D_i/r_i + D_j/r_j - D_k/r_k)/\sqrt{r} \\ C_{ijk} &= (D_i^2/r_i + D_j^2/r_j - D_k^2/r_k)/2. \end{aligned} \quad (4.6)$$

The relations (4.6) show that the function $G_{mnij}(z)$ and its derivatives are bounded on $(-\infty, \infty)$. So, the remainder (4.3) for this function goes to zero as N becomes large, and the numerical value of $G_{mn}(g_i, g_j)$ can be approximated by $\sum_{\ell=1}^N w_{\ell} G_{mnij}(z_{\ell})$. However, it is noticed that when r is very small the contribution of $G_{mnij}(z_{\ell})$ to the summation, for large values of z_{ℓ} , may be very trivial due to the largeness of the exponential terms $\exp Q_{ijk}(z_{\ell})$. This causes a slow convergence and sometimes it leads to poor approximations. To obtain more accurate results, we apply the Romberg's algorithm, which is known [10], for the numerical evaluation of (4.5) when r is small. This can be done by breaking the range of integration and using some appropriate transformations to change the integral (4.5) into sum of three integrals on the interval $(0, 1)$.

The function $G_{mnij}(z)$ is so complicated that we cannot find an estimate for the remainder term though the computation shows that our results are accurate at least up to four decimal points. To have a better check on the

results, in the next section we introduce some power series for the numerical evaluation of $G_{mn}(g_i, g_j)$.

5. SOME POWER SERIES EXPANSIONS FOR THE COMPUTATION OF

$$G_{mn}(g_i, g_j)$$

In this section, we mention briefly some power series expansions for the evaluation of the integral (3.7), and we compare some of the results with the above approximations.

First we use a series expansion similar to the one employed by Hill [8] for the computation of $I(p, p)$, in the special case that $\sigma_1^2 = \sigma_2^2$. We observe, from (3.4)-(3.5), that

$$\frac{g_i(y)g_j(y)}{g(y)} = \frac{1}{\sqrt{2\pi r_i r_j}/r} \frac{h_i(y)h_j(y)}{h(y)}, \quad (5.1)$$

where

$$h(y) = ph_1(y) + qrh_2(y) \quad (5.2)$$

with

$$h_i(y) = \exp[-(y-D_i)^2/2r_i], \quad i = 1, 2. \quad (5.3)$$

Now, it is easy to show that $ph_1(y)/qrh_2(y) < 1$ if y is in the interval $(-\infty, \alpha_1)$ or (α_2, ∞) , with $\alpha_1 < \alpha_2$, and $qrh_2(y)/ph_1(y) < 1$ if y is in the interval (α_1, α_2) , where α_1 and α_2 are the real roots of the equation

$$(1-r^2)y^2 + 2D(1+r^2)y + D^2(1-r^2) - 2r \log(p/qr) = 0, \quad (5.4)$$

if such real roots exist. When the roots are imaginary or equal,

$ph_1(y)/qrh_2(y) < 1$ for all y . If $r = 1$, then $ph_1(y)/qrh_2(y) < 1$ for y

in the interval (α, ∞) and $qrh_2(y)/ph(y) < 1$ for y in the interval $(-\infty, \alpha)$, where $\alpha = \log(p/q)/2D$. Dividing the numerator and denominator of (5.1) by $qrh_2(y)$ or $ph_1(y)$, breaking the real line into appropriate intervals, using geometric expansions, we obtain

$$G_{mn}(g_i, g_j) = \frac{1}{\sqrt{2\pi r_1 r_2 / r}} \sum_{N=0}^{\infty} \left[\int_{-\infty}^{\alpha_1} H_N(y) dy + \int_{\alpha_1}^{\alpha_2} \bar{H}_N(y) dy + \int_{\alpha_2}^{\infty} H_N(y) dy \right], \quad (5.5)$$

where

$$\begin{aligned} H_N(y) &= (y-D_1)^m (y-D_2)^n \phi_N(y) \\ \bar{H}_N(y) &= (y-D_1)^m (y-D_2)^n \bar{\phi}_N(y) \end{aligned} \quad (5.6)$$

with

$$\begin{aligned} \phi_N(y) &= \frac{1}{qr} \left(-\frac{p}{qr} \right)^N h_1(y) h_2(y) h_2^{-1}(y) [h_1(y)/h_2(y)]^N \\ \bar{\phi}_N(y) &= \frac{1}{p} \left(\frac{-qr}{p} \right)^N h_1(y) h_2(y) h_1^{-1}(y) [h_2(y)/h_1(y)]^N. \end{aligned} \quad (5.7)$$

It follows from (5.2)-(5.3) and (5.7) that $\phi_N(y)$ and $\bar{\phi}_N(y)$ are constant multiples of normal densities. Thus, the computation of the integrals in (5.5) leads to computation of truncated moments of a normal distribution, which is already known [5]. The advantage of the method is the fact that we can approximate $G_{mn}(g_i, g_j)$ by an alternating series and estimate the error, although the computation is rather heavy.

When $r < 1$, the equation (5.4) has imaginary or equal roots if and only if

$$D^2 \leq (1-r^2) \log(qr/p)/2r. \quad (5.8)$$

In this particular case, we have only the integrals $\int_{-\infty}^{\infty} H_N(y) dy$ in (5.5) to evaluate. As an example, after doing the necessary calculation, we obtain the following expression for $G_{00}(g_1, g_2)$ when (5.8) holds:

$$G_{00}(g_1, g_2) = \frac{1}{rq} \sum_{N=0}^{\infty} (-1)^N r a_N^{-1/2} \exp[2D^2 r N(N+1)/a_N - \alpha N/2r], \quad (5.9)$$

where

$$a_N = N(1-r^2) + 1, \quad N = 0, 1, \dots \quad (5.10)$$

Now, the information about p becomes

$$I(p, p) = [1 - G_{00}(g_1, g_2)]/pq. \quad (5.11)$$

Another power series, which leads to calculation of the first few moments of normal densities, may be obtained by using the fact that $0 < h(y) < 1$ for all y in $(-\infty, \infty)$ and expanding $1/h(y) = 1/[1-h(y)]$. For this series, the calculation is simpler, but the convergence is slow.

Finally, when the mixed densities are not well separated, i.e., D is close to zero and r close to 1, calculation shows that the Taylor approximation of $g_i(y)g_j(y)/g(y)$, as a function of D and r , in the neighborhood of the point $(0, 1)$ is of the form $P_{ij}(y)\phi(y)$, where $P_{ij}(y)$ is a polynomial whose coefficients depend on D , p , r and $\phi(y)$ is the standard normal density. Now it is easy to find an approximate value for $G_{mn}(g_1, g_j)$. As an example, we have found the following expression for $I(p, p)$ by using the first six terms of the Taylor series when D is close to zero and r close to one:

$$I(p, p) \approx 4D^2 + 2(1-r)^2 + 12(2p-1)(1-r)D^2 + 2(1-r)^3(4p-1). \quad (5.12)$$

To compare the above numerical methods, we have computed $I(p, p)$ for some values of D , p , r by the Hermite-Gauss quadrature with 14 terms, by the Romberg's algorithm with 8 iterations, by the power series with 25 terms, and by the Taylor series with 6 terms. The results are given for comparison as follows:

Parameters			The Values of $I(p,p)$			
D	p	r	Hermite-Gauss	Romberg	Power Series	Taylor
.05	.3	.95	.01438	0.01431	.01438	.01445
.06	.3	.94	.02051	0.02044	.02051	.02065
.07	.3	.93	.02765	0.02757	.02764	.02789
.08	.3	.92	.03576	0.03568	.03575	.03614

We observe that all these methods agree with each other up to three or four decimal figures.

We have provided a brief table of the elements of the standard information matrix for some values of D , p , r , which may be useful in practice for finding the information matrix about all or some of the parameters.

There are two special cases which should be considered separately:

(1) A mixture of two normal densities with equal variances σ^2 and different means μ_1 and μ_2 , i.e., when $r = 1$ and $D \neq 0$

(2) A mixture of two normal densities with equal means μ and different variances σ_1^2 and σ_2^2 , i.e., when $D = 0$ and $r \neq 1$.

In these two cases the information matrices are 4×4 and it is easy to verify, by using the relations (2.1) and (2.2), that these matrices can be obtained from the 5×5 information matrix I according to the following rules:

In the first case, we add termwise first the last two rows and then the last two columns of I to obtain the information matrix about $\mu_1, \mu_2, p, \sigma^2$.

In the second case, we add termwise first the first two rows and then the first two columns of I to obtain the information matrix about $\mu, p, \sigma_1^2, \sigma_2^2$.

We observe from the relations (2.4), that when D goes to zero and r goes to one, that is when the mixed densities become closer and closer to each other, the information matrix I approaches to a singular matrix continuously, with some diagonal elements equal to zero. The same assertion is true when p goes to one or zero.

It is known that for a sample of size, say N , I^{-1}/N is the asymptotic covariance matrix of the maximum likelihood estimates of the parameters, and the diagonal elements of I^{-1}/N are the variances of the asymptotic distribution of these estimates. Combining this fact with the above assertion, we conclude that for estimating the parameters of a mixture of two normal distributions with mixed densities, which are not well-separated, or with a mixing proportion close to zero we need a huge sample, and the estimation may be impossible.

We now look at a numerical example. Consider a mixture of two normal distributions with $\mu_1 = 6\sqrt{5}$, $\mu_2 = 2\sqrt{5}$, $p = .5$, $\sigma_1 = 4$, $\sigma_2 = 5$. Since $D = 1$, $p = .5$, $r = .8$ and $\mu_1 > \mu_2$, from the table we obtain the symmetric positive definite matrix

$$J = \begin{bmatrix} 0.3252 & -0.0402 & 0.4026 & 0.0957 & 0.0461 \\ & 0.346 & 0.4864 & -0.0212 & -0.1016 \\ & & 2.1923 & -0.1237 & 0.1448 \\ & & & 0.1511 & -0.0297 \\ & & & & 0.1517 \end{bmatrix} .$$

Using $I = \Delta J \Delta$ with $\Delta = \text{diag}(1/4, 1/5, 1, 1/16, 1/25)$, we have the information matrix

$$I = \begin{bmatrix} 0.0233 & -0.0018 & 0.1007 & 0.0015 & 0.0005 \\ & 0.0126 & 0.0973 & -0.0003 & -0.0008 \\ & & 2.1923 & -0.0077 & 0.0058 \\ & & & 0.0006 & 0.0000 \\ & & & & 0.0002 \end{bmatrix} .$$

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SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO NORMAL DISTRIBUTIONS

$\tau = 0.2$

P	D	J(1,1)	J(1,2)	J(1,3)	J(1,4)	J(1,5)	J(2,2)	J(2,3)	J(2,4)	J(2,5)	J(3,3)	J(3,4)	J(3,5)	J(4,4)	J(4,5)	J(5,5)
0.1	0.0 *	0.0157	0.0169	0.0	0.0	0.0	0.8963	0.0	0.0	0.0	2.1540	-0.0650	-0.3843	0.0055	-0.0010	0.4328
	0.2 *	0.0159	0.0167	-0.0049	-0.0002	-0.0032	0.8940	-0.1426	-0.0010	-0.0056	2.1925	-0.0660	-0.3700	0.0056	-0.0010	0.4338
	0.4 *	0.0165	0.0164	-0.0101	-0.0004	-0.0063	0.8872	-0.2814	-0.0021	-0.0099	2.3079	-0.0677	-0.3276	0.0058	-0.0008	0.4365
	0.6 *	0.0176	0.0157	-0.0159	-0.0006	-0.0093	0.8763	-0.4124	-0.0033	-0.0109	2.5073	-0.0706	-0.2591	0.0062	-0.0004	0.4402
	0.8 *	0.0192	0.0147	-0.0224	-0.0009	-0.0120	0.8620	-0.5309	-0.0046	-0.0090	2.7972	-0.0747	-0.1678	0.0067	0.0001	0.4435
	1.0 *	0.0214	0.0133	-0.0301	-0.0012	-0.0142	0.8453	-0.6322	-0.0062	-0.0028	3.1823	-0.0799	-0.0593	0.0075	0.0009	0.4451
0.2	0.0 *	0.0558	0.0288	0.0	0.0	0.0	0.7939	0.0	0.0	0.0	1.9493	-0.1064	-0.3261	0.0192	-0.0049	0.3746
	0.2 *	0.0564	0.0285	-0.0080	-0.0007	-0.0057	0.7905	-0.1206	-0.0029	-0.0079	1.9785	-0.1069	-0.3131	0.0194	-0.0047	0.3761
	0.4 *	0.0582	0.0274	-0.0162	-0.0014	-0.0112	0.7806	-0.2362	-0.0059	-0.0141	2.0652	-0.1083	-0.2754	0.0200	-0.0041	0.3803
	0.6 *	0.0613	0.0256	-0.0250	-0.0021	-0.0162	0.7650	-0.3417	-0.0091	-0.0155	2.2130	-0.1106	-0.2153	0.0211	-0.0030	0.3859
	0.8 *	0.0656	0.0230	-0.0342	-0.0031	-0.0203	0.7452	-0.4319	-0.0124	-0.0122	2.4177	-0.1134	-0.1377	0.0226	-0.0014	0.3910
	1.0 *	0.0714	0.0195	-0.0437	-0.0042	-0.0234	0.7232	-0.5024	-0.0158	-0.0031	2.6784	-0.1163	-0.0489	0.0246	0.0008	0.3948
0.3	0.0 *	0.1145	0.0371	0.0	0.0	0.0	0.6923	0.0	0.0	0.0	1.8633	-0.1373	-0.2867	0.0392	-0.0094	0.3210
	0.2 *	0.1155	0.0359	-0.0102	-0.0014	-0.0075	0.6884	-0.1059	-0.0049	-0.0090	1.8879	-0.1375	-0.2749	0.0395	-0.0090	0.3228
	0.4 *	0.1185	0.0348	-0.0206	-0.0028	-0.0147	0.6772	-0.2064	-0.0098	-0.0157	1.9601	-0.1380	-0.2407	0.0406	-0.0079	0.3276
	0.6 *	0.1235	0.0318	-0.0311	-0.0043	-0.0210	0.6598	-0.2958	-0.0148	-0.0171	2.0816	-0.1389	-0.1869	0.0424	-0.0060	0.3340
	0.8 *	0.1305	0.0276	-0.0419	-0.0061	-0.0260	0.6383	-0.3697	-0.0197	-0.0131	2.2462	-0.1396	-0.1187	0.0450	-0.0032	0.3398
	1.0 *	0.1395	0.0223	-0.0523	-0.0081	-0.0293	0.6150	-0.4238	-0.0246	-0.0028	2.4499	-0.1396	-0.0426	0.0484	0.0003	0.3437
0.4	0.0 *	0.1887	0.0423	0.0	0.0	0.0	0.5913	0.0	0.0	0.0	1.8657	-0.1641	-0.2585	0.0650	-0.0135	0.2704
	0.2 *	0.1900	0.0415	-0.0122	-0.0022	-0.0088	0.5874	-0.0940	-0.0068	-0.0091	1.8874	-0.1639	-0.2477	0.0655	-0.0129	0.2722
	0.4 *	0.1941	0.0391	-0.0244	-0.0045	-0.0170	0.5760	-0.1851	-0.0131	-0.0153	1.9533	-0.1635	-0.2162	0.0671	-0.0113	0.2771
	0.6 *	0.2007	0.0352	-0.0364	-0.0069	-0.0241	0.5586	-0.2638	-0.0196	-0.0169	2.0618	-0.1627	-0.1671	0.0697	-0.0086	0.2836
	0.8 *	0.2099	0.0298	-0.0481	-0.0095	-0.0294	0.5373	-0.3270	-0.0257	-0.0127	2.2066	-0.1609	-0.1058	0.0734	-0.0049	0.2901
	1.0 *	0.2215	0.0230	-0.0590	-0.0123	-0.0326	0.5149	-0.3711	-0.0314	-0.0023	2.3827	-0.1579	-0.0385	0.0782	-0.0004	0.2932
0.5	0.0 *	0.2769	0.0446	0.0	0.0	0.0	0.4909	0.0	0.0	0.0	1.9560	-0.1906	-0.2377	0.0970	-0.0168	0.2218
	0.2 *	0.2783	0.0437	-0.0142	-0.0031	-0.0095	0.4872	-0.0876	-0.0080	-0.0085	1.9771	-0.1900	-0.2275	0.0978	-0.0161	0.2235
	0.4 *	0.2833	0.0408	-0.0284	-0.0063	-0.0182	0.4764	-0.1695	-0.0156	-0.0142	2.0420	-0.1886	-0.1981	0.0997	-0.0140	0.2282
	0.6 *	0.2916	0.0362	-0.0419	-0.0095	-0.0256	0.4602	-0.2342	-0.0238	-0.0156	2.1456	-0.1858	-0.1528	0.1029	-0.0108	0.2347
	0.8 *	0.3019	0.0299	-0.0542	-0.0130	-0.0309	0.4406	-0.2962	-0.0299	-0.0114	2.2834	-0.1816	-0.0966	0.1079	-0.0063	0.2402
	1.0 *	0.3153	0.0223	-0.0654	-0.0165	-0.0337	0.4207	-0.3338	-0.0360	-0.0017	2.4488	-0.1756	-0.0357	0.1140	-0.0010	0.2428
0.6	0.0 *	0.3789	0.0442	0.0	0.0	0.0	0.3910	0.0	0.0	0.0	2.1629	-0.2201	-0.2220	0.1361	-0.0188	0.1749
	0.2 *	0.3807	0.0432	-0.0163	-0.0042	-0.0095	0.3877	-0.0819	-0.0086	-0.0074	2.1846	-0.2190	-0.2125	0.1369	-0.0180	0.1764
	0.4 *	0.3859	0.0400	-0.0322	-0.0080	-0.0182	0.3782	-0.1579	-0.0170	-0.0123	2.2525	-0.2163	-0.1848	0.1394	-0.0157	0.1808
	0.6 *	0.3944	0.0350	-0.0475	-0.0121	-0.0254	0.3640	-0.2232	-0.0248	-0.0134	2.3604	-0.2116	-0.1423	0.1435	-0.0121	0.1862
	0.8 *	0.4059	0.0284	-0.0609	-0.0162	-0.0304	0.3470	-0.2736	-0.0319	-0.0096	2.5023	-0.2046	-0.0900	0.1492	-0.0073	0.1910
	1.0 *	0.4200	0.0204	-0.0726	-0.0202	-0.0328	0.3299	-0.3065	-0.0379	-0.0011	2.6711	-0.1955	-0.0339	0.1564	-0.0016	0.1932
0.7	0.0 *	0.4954	0.0409	0.0	0.0	0.0	0.2917	0.0	0.0	0.0	2.5689	-0.2569	-0.2108	0.1841	-0.0193	0.1295
	0.2 *	0.4972	0.0398	-0.0190	-0.0048	-0.0089	0.2890	-0.0777	-0.0086	-0.0059	2.5949	-0.2554	-0.2016	0.1850	-0.0185	0.1307
	0.4 *	0.5024	0.0367	-0.0374	-0.0095	-0.0170	0.2813	-0.1495	-0.0170	-0.0098	2.6728	-0.2511	-0.1751	0.1878	-0.0161	0.1341
	0.6 *	0.5109	0.0317	-0.0575	-0.0141	-0.0239	0.2697	-0.2106	-0.0247	-0.0106	2.7963	-0.2439	-0.1348	0.1924	-0.0124	0.1384
	0.8 *	0.5222	0.0251	-0.0694	-0.0187	-0.0280	0.2562	-0.2572	-0.0314	-0.0074	2.9578	-0.2338	-0.0855	0.1988	-0.0076	0.1421
	1.0 *	0.5360	0.0174	-0.0818	-0.0230	-0.0298	0.2425	-0.2868	-0.0369	-0.0005	3.1481	-0.2209	-0.0329	0.2068	-0.0021	0.1439
0.8	0.0 *	0.6287	0.0343	0.0	0.0	0.0	0.1931	0.0	0.0	0.0	3.4378	-0.3103	-0.2036	0.2446	-0.0176	0.0850
	0.2 *	0.6303	0.0333	-0.0230	-0.0052	-0.0076	0.1904	-0.0749	-0.0078	-0.0041	3.4731	-0.3082	-0.1947	0.2456	-0.0169	0.0863
	0.4 *	0.6350	0.0304	-0.0451	-0.0103	-0.0144	0.1856	-0.1440	-0.0152	-0.0069	3.5534	-0.2999	-0.1710	0.2485	-0.0146	0.0882
	0.6 *	0.6426	0.0259	-0.0649	-0.0152	-0.0198	0.1774	-0.2025	-0.0220	-0.0073	3.7402	-0.2912	-0.1303	0.2534	-0.0114	0.0912
	0.8 *	0.6527	0.0201	-0.0819	-0.0198	-0.0232	0.1677	-0.2463	-0.0277	-0.0050	3.9549	-0.2768	-0.0829	0.2600	-0.0072	0.0939
	1.0 *	0.6648	0.0134	-0.0952	-0.0240	-0.0245	0.1581	-0.2737	-0.0323	0.0001	4.2045	-0.2590	-0.0329	0.2682	-0.0023	0.0951
0.9	0.0 *	0.7852	0.0230	0.0	0.0	0.0	0.0954	0.0	0.0	0.0	6.0560	-0.4104	-0.2019	0.3270	-0.0128	0.0418
	0.2 *	0.7863	0.0222	-0.0303	-0.0049	-0.0051	0.0943	-0.0743	-0.0056	-0.0021	6.1216	-0.4070	-0.1932	0.3279	-0.0123	0.0422
	0.4 *	0.7897	0.0202	-0.0590	-0.0096	-0.0097	0.0914	-0.1427	-0.0109	-0.0035	6.3144	-0.3970	-0.1679	0.3306	-0.0108	0.0434
	0.6 *	0.7951	0.0169	-0.0845	-0.0140	-0.0133	0.0869	-0.2001	-0.0156	-0.0037	6.6170	-0.3805	-0.1298	0.3349	-0.0084	0.0449
	0.8 *	0.8022	0.0127	-0.1056	-0.0180	-0.0154	0.0818	-0.2428	-0.0195	-0.0023	7.0128	-0.3587	-0.0834	0.3408	-0.0054	0.0463
	1.0 *	0.8107	0.0081	-0.1211	-0.0213	-0.0160	0.0767	-0.2689	-0.0225	0.0004	7.4696	-0.3322	-0.0346	0.3479	-0.0021	0.0469

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO NORMAL DISTRIBUTIONS

r = 0.4

P	D	J(1,1)	J(1,2)	J(1,3)	J(1,4)	J(1,5)	J(2,2)	J(2,3)	J(2,4)	J(2,5)	J(3,3)	J(3,4)	J(3,5)	J(4,4)	J(4,5)	J(5,5)
0.1	0.0	* 0.0096	0.0361	0.0	0.0	0.0	0.8855	0.0	0.0	0.0	0.8245	-0.0388	-0.3826	0.0035	0.0060	0.4346
	0.2	* 0.0100	0.0358	-0.0097	-0.0003	-0.0094	0.8805	-0.2290	-0.0008	-0.0042	0.8829	-0.0399	-0.3518	0.0036	0.0060	0.4358
	0.4	* 0.0110	0.0346	-0.0208	-0.0008	-0.0184	0.8657	-0.4491	-0.0016	-0.0045	1.0652	-0.0422	-0.2615	0.0040	0.0061	0.4384
	0.6	* 0.0130	0.0324	-0.0351	-0.0013	-0.0266	0.8430	-0.6497	-0.0026	0.0026	1.3932	-0.0461	-0.1187	0.0047	0.0063	0.4393
	0.8	* 0.0161	0.0286	-0.0544	-0.0022	-0.0331	0.8142	-0.8171	-0.0038	0.0190	1.8999	-0.0514	0.0624	0.0058	0.0066	0.4350
	1.0	* 0.0208	0.0225	-0.0802	-0.0036	-0.0367	0.7847	-0.9336	-0.0052	0.0442	2.6276	-0.0581	0.2588	0.0077	0.0070	0.4214
0.2	0.0	* 0.0374	0.0650	0.0	0.0	0.0	0.7740	0.0	0.0	0.0	0.8258	-0.0742	-0.3527	0.0132	0.0089	0.3749
	0.2	* 0.0385	0.0639	-0.0182	-0.0014	-0.0173	0.7660	-0.2100	-0.0025	-0.0062	0.8802	-0.0751	-0.3232	0.0136	0.0090	0.3768
	0.4	* 0.0419	0.0601	-0.0383	-0.0030	-0.0333	0.7431	-0.4059	-0.0051	-0.0060	1.0464	-0.0778	-0.2384	0.0149	0.0094	0.3806
	0.6	* 0.0479	0.0533	-0.0618	-0.0051	-0.0467	0.7092	-0.5725	-0.0079	0.0054	1.3322	-0.0819	-0.1095	0.0171	0.0100	0.3819
	0.8	* 0.0569	0.0427	-0.0898	-0.0080	-0.0553	0.6706	-0.6936	-0.0108	0.0292	1.7442	-0.0867	0.0438	0.0207	0.0107	0.3752
	1.0	* 0.0691	0.0278	-0.1214	-0.0120	-0.0569	0.6361	-0.7553	-0.0137	0.0622	2.2780	-0.0914	0.1947	0.0257	0.0114	0.3575
0.3	0.0	* 0.0821	0.0872	0.0	0.0	0.0	0.6652	0.0	0.0	0.0	0.8465	-0.1077	-0.3282	0.0287	0.0099	0.3195
	0.2	* 0.0841	0.0849	-0.0262	-0.0030	-0.0235	0.6557	-0.1947	-0.0045	-0.0065	0.8993	-0.1084	-0.3000	0.0295	0.0101	0.3216
	0.4	* 0.0903	0.0781	-0.0540	-0.0065	-0.0447	0.6292	-0.3720	-0.0091	-0.0055	1.0578	-0.1103	-0.2204	0.0319	0.0108	0.3257
	0.6	* 0.1009	0.0662	-0.0844	-0.0108	-0.0611	0.5915	-0.5147	-0.0137	0.0080	1.3209	-0.1128	-0.1029	0.0361	0.0118	0.3267
	0.8	* 0.1158	0.0488	-0.1170	-0.0163	-0.0697	0.5513	-0.6079	-0.0179	0.0344	1.6812	-0.1150	0.0306	0.0424	0.0129	0.3191
	1.0	* 0.1348	0.0265	-0.1489	-0.0229	-0.0680	0.5187	-0.6426	-0.0216	0.0682	2.1186	-0.1158	0.1541	0.0508	0.0135	0.3011
0.4	0.0	* 0.1434	0.1027	0.0	0.0	0.0	0.5590	0.0	0.0	0.0	0.8902	-0.1415	-0.3076	0.0502	0.0097	0.2674
	0.2	* 0.1463	0.0994	-0.0343	-0.0053	-0.0280	0.5492	-0.1820	-0.0065	-0.0059	0.9434	-0.1417	-0.2809	0.0514	0.0100	0.2694
	0.4	* 0.1552	0.0895	-0.0694	-0.0112	-0.0526	0.5224	-0.3445	-0.0129	-0.0040	1.1011	-0.1422	-0.2061	0.0550	0.0110	0.2731
	0.6	* 0.1698	0.0729	-0.1055	-0.0179	-0.0703	0.4853	-0.4698	-0.0187	0.0102	1.3559	-0.1422	-0.0983	0.0613	0.0122	0.2736
	0.8	* 0.1898	0.0499	-0.1406	-0.0259	-0.0778	0.4477	-0.5448	-0.0238	0.0361	1.6916	-0.1409	0.0203	0.0703	0.0134	0.2659
	1.0	* 0.2139	0.0225	-0.1711	-0.0349	-0.0730	0.4196	-0.5648	-0.0278	0.0673	2.0809	-0.1374	0.1256	0.0818	0.0138	0.2494
0.5	0.0	* 0.2213	0.1115	0.0	0.0	0.0	0.4554	0.0	0.0	0.0	0.9646	-0.1779	-0.2903	0.0781	0.0088	0.2181
	0.2	* 0.2251	0.1073	-0.0429	-0.0080	-0.0307	0.4463	-0.1713	-0.0080	-0.0045	1.0208	-0.1774	-0.2649	0.0797	0.0092	0.2197
	0.4	* 0.2361	0.0947	-0.0855	-0.0165	-0.0570	0.4214	-0.3218	-0.0157	-0.0018	1.1852	-0.1759	-0.1946	0.0847	0.0103	0.2226
	0.6	* 0.2540	0.0744	-0.1268	-0.0259	-0.0747	0.3879	-0.4340	-0.0224	0.0118	1.4451	-0.1727	-0.0952	0.0929	0.0116	0.2223
	0.8	* 0.2775	0.0475	-0.1639	-0.0362	-0.0805	0.3553	-0.4967	-0.0279	0.0351	1.7776	-0.1673	0.0115	0.1043	0.0125	0.2151
	1.0	* 0.3047	0.0173	-0.1923	-0.0466	-0.0731	0.3324	-0.5080	-0.0318	0.0621	2.1500	-0.1594	0.1039	0.1183	0.0126	0.2011
0.6	0.0	* 0.3171	0.1132	0.0	0.0	0.0	0.3547	0.0	0.0	0.0	1.0869	-0.2198	-0.2753	0.1137	0.0076	0.1710
	0.2	* 0.3213	0.1083	-0.0527	-0.0109	-0.0314	0.3468	-0.1621	-0.0089	-0.0027	1.1495	-0.2185	-0.2528	0.1158	0.0082	0.1721
	0.4	* 0.3337	0.0939	-0.1038	-0.0222	-0.0576	0.3257	-0.3029	-0.0172	0.0005	1.3311	-0.2145	-0.1854	0.1218	0.0090	0.1739
	0.6	* 0.3533	0.0712	-0.1507	-0.0340	-0.0742	0.2978	-0.4049	-0.0242	0.0127	1.6129	-0.2075	-0.0934	0.1318	0.0100	0.1729
	0.8	* 0.3784	0.0426	-0.1894	-0.0460	-0.0781	0.2715	-0.4590	-0.0295	0.0321	1.9650	-0.1974	0.0035	0.1451	0.0105	0.1666
	1.0	* 0.4067	0.0119	-0.2156	-0.0571	-0.0690	0.2540	-0.4653	-0.0333	0.0538	2.3495	-0.1847	0.0861	0.1608	0.0102	0.1556
0.7	0.0	* 0.4327	0.1069	0.0	0.0	0.0	0.2572	0.0	0.0	0.0	1.2968	-0.2726	-0.2620	0.1594	0.0065	0.1259
	0.2	* 0.4370	0.1017	-0.0651	-0.0139	-0.0297	0.2511	-0.1541	-0.0088	-0.0008	1.3720	-0.2702	-0.2396	0.1617	0.0067	0.1264
	0.4	* 0.4496	0.0867	-0.1265	-0.0276	-0.0540	0.2349	-0.2866	-0.0169	0.0027	1.5889	-0.2630	-0.1781	0.1687	0.0073	0.1270
	0.6	* 0.4691	0.0636	-0.1801	-0.0413	-0.0685	0.2138	-0.3808	-0.0235	0.0125	1.9206	-0.2513	-0.0932	0.1797	0.0077	0.1254
	0.8	* 0.4936	0.0356	-0.2209	-0.0542	-0.0706	0.1946	-0.4290	-0.0285	0.0273	2.3275	-0.2359	-0.0046	0.1940	0.0076	0.1203
	1.0	* 0.5204	0.0068	-0.2448	-0.0649	-0.0609	0.1823	-0.4329	-0.0319	0.0431	2.7634	-0.2177	0.0704	0.2103	0.0069	0.1124
0.8	0.0	* 0.5725	0.0910	0.0	0.0	0.0	0.1636	0.0	0.0	0.0	1.7063	-0.3475	-0.2497	0.2198	0.0055	0.0823
	0.2	* 0.5764	0.0861	-0.0826	-0.0160	-0.0252	0.1596	-0.1469	-0.0075	0.0008	1.8096	-0.3436	-0.2290	0.2222	0.0055	0.0824
	0.4	* 0.5875	0.0721	-0.1586	-0.0315	-0.0454	0.1491	-0.2721	-0.0143	0.0042	2.1048	-0.3324	-0.1726	0.2301	0.0054	0.0819
	0.6	* 0.6046	0.0512	-0.2216	-0.0460	-0.0567	0.1357	-0.3605	-0.0198	0.0111	2.5512	-0.3148	-0.0950	0.2404	0.0049	0.0800
	0.8	* 0.6255	0.0268	-0.2657	-0.0585	-0.0574	0.1237	-0.4052	-0.0240	0.0206	3.0913	-0.2926	-0.0142	0.2543	0.0040	0.0764
	1.0	* 0.6481	0.0026	-0.2872	-0.0677	-0.0485	0.1162	-0.4087	-0.0271	0.0305	3.6629	-0.2677	0.0548	0.2696	0.0029	0.0716
0.9	0.0	* 0.7462	0.0615	0.0	0.0	0.0	0.0754	0.0	0.0	0.0	2.8245	-0.4814	-0.2362	0.3067	0.0047	0.0403
	0.2	* 0.7488	0.0579	-0.1140	-0.0159	-0.0167	0.0737	-0.1388	-0.0045	0.0017	3.0143	-0.4756	-0.2182	0.3087	0.0044	0.0400
	0.4	* 0.7562	0.0476	-0.2156	-0.0308	-0.0298	0.0692	-0.2576	-0.0086	0.0043	3.5553	-0.4575	-0.1690	0.3145	0.0035	0.0390
	0.6	* 0.7675	0.0328	-0.2962	-0.0436	-0.0368	0.0635	-0.3422	-0.0121	0.0078	4.3681	-0.4329	-0.1008	0.3234	0.0019	0.0373
	0.8	* 0.7811	0.0159	-0.3471	-0.0536	-0.0367	0.0584	-0.3864	-0.0151	0.0121	5.3446	-0.4011	-0.0286	0.3343	0.0010	0.0354
	1.0	* 0.7958	-0.0000	-0.3664	-0.0599	-0.0306	0.0553	-0.3928	-0.0177	0.0162	6.3734	-0.3658	0.0352	0.3458	-0.0011	0.0335

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO NORMAL DISTRIBUTIONS

r = 0.6

P	D	J(1,1)	J(1,2)	J(1,3)	J(1,4)	J(1,5)	J(2,2)	J(2,3)	J(2,4)	J(2,5)	J(3,3)	J(3,4)	J(3,5)	J(4,4)	J(4,5)	J(5,5)	
0.1	0.0	*	0.0079	0.0552	0.0	0.0	0.8668	0.0	0.0	0.0	0.3068	-0.0253	-0.3020	0.0029	0.0162	0.4355	
	0.2	*	0.0085	0.0545	-0.0154	-0.0007	-0.0171	0.8592	-0.2898	-0.0003	0.0032	0.3873	-0.0258	-0.2561	0.0031	0.0161	0.4348
	0.4	*	0.0104	0.0518	-0.0352	-0.0016	-0.0332	0.8375	-0.5629	-0.0005	0.0131	0.6393	-0.0268	-0.1241	0.0039	0.0157	0.4306
	0.6	*	0.0141	0.0462	-0.0644	-0.0032	-0.0461	0.8056	-0.7972	-0.0004	0.0339	1.1199	-0.0277	0.0748	0.0055	0.0146	0.4183
	0.8	*	0.0204	0.0356	-0.1084	-0.0059	-0.0527	0.7714	-0.9614	0.0004	0.0651	1.9090	-0.0278	0.3004	0.0084	0.0117	0.3945
	1.0	*	0.0298	0.0188	-0.1674	-0.0100	-0.0484	0.7462	-1.0222	0.0022	0.0976	3.0594	-0.0276	0.4920	0.0131	0.0062	0.3638
0.2	0.0	*	0.0318	0.1009	0.0	0.0	0.7394	0.0	0.0	0.0	0.3214	-0.0513	-0.2851	0.0116	0.0292	0.3749	
	0.2	*	0.0337	0.0982	-0.0308	-0.0026	-0.0313	0.7271	-0.2715	-0.0010	0.0068	0.4003	-0.0517	-0.2417	0.0124	0.0288	0.3735
	0.4	*	0.0398	0.0894	-0.0675	-0.0061	-0.0587	0.6953	-0.5151	-0.0015	0.0233	0.6463	-0.0523	-0.1210	0.0150	0.0272	0.3659
	0.6	*	0.0509	0.0724	-0.1147	-0.0113	-0.0770	0.6524	-0.7000	-0.0010	0.0533	1.0801	-0.0524	0.0475	0.0201	0.0234	0.3478
	0.8	*	0.0671	0.0456	-0.1722	-0.0187	-0.0798	0.6070	-0.7973	0.0009	0.0908	1.7083	-0.0521	0.2162	0.0282	0.0157	0.3233
	1.0	*	0.0863	0.0115	-0.2289	-0.0274	-0.0636	0.5958	-0.7960	0.0030	0.1217	2.4845	-0.0542	0.3369	0.0386	0.0043	0.2919
0.3	0.0	*	0.0720	0.1368	0.0	0.0	0.6179	0.0	0.0	0.0	0.3382	-0.0785	-0.2690	0.0262	0.0393	0.3178	
	0.2	*	0.0756	0.1316	-0.0466	-0.0059	-0.0422	0.6040	-0.2546	-0.0015	0.0105	0.4198	-0.0784	-0.2285	0.0279	0.0384	0.3155
	0.4	*	0.0867	0.1151	-0.0985	-0.0132	-0.0772	0.5692	-0.4739	-0.0022	0.0309	0.6663	-0.0778	-0.1187	0.0330	0.0351	0.3055
	0.6	*	0.1054	0.0860	-0.1578	-0.0229	-0.0966	0.5264	-0.6250	-0.0011	0.0631	1.0751	-0.0763	0.0261	0.0422	0.0278	0.2855
	0.8	*	0.1297	0.0453	-0.2183	-0.0347	-0.0936	0.4939	-0.6874	0.0013	0.0979	1.6174	-0.0752	0.1605	0.0553	0.0154	0.2598
	1.0	*	0.1553	0.0006	-0.2649	-0.0461	-0.0690	0.4829	-0.6660	0.0022	0.1229	2.2261	-0.0783	0.2506	0.0695	0.0006	0.2394
0.4	0.0	*	0.1292	0.1625	0.0	0.0	0.5020	0.0	0.0	0.0	0.3615	-0.1078	-0.2534	0.0473	0.0467	0.2640	
	0.2	*	0.1345	0.1546	-0.0633	-0.0103	-0.0498	0.4895	-0.2386	-0.0018	0.0138	0.4478	-0.1070	-0.2160	0.0499	0.0451	0.2604
	0.4	*	0.1502	0.1305	-0.1299	-0.0223	-0.0888	0.4566	-0.4373	-0.0022	0.0359	0.7021	-0.1046	-0.1171	0.0578	0.0397	0.2491
	0.6	*	0.1750	0.0908	-0.1980	-0.0366	-0.1070	0.4196	-0.5646	-0.0006	0.0661	1.1035	-0.1011	0.0083	0.0709	0.0288	0.2298
	0.8	*	0.2048	0.0406	-0.2574	-0.0518	-0.0988	0.3948	-0.6080	0.0016	0.0950	1.6034	-0.0991	0.1199	0.0877	0.0126	0.2091
	1.0	*	0.2337	-0.0093	-0.2932	-0.0643	-0.0695	0.3896	-0.5808	0.0006	0.1139	2.1315	-0.1021	0.1941	0.1041	-0.0036	0.1957
0.5	0.0	*	0.2044	0.1774	0.0	0.0	0.3931	0.0	0.0	0.0	0.3941	-0.1406	-0.2378	0.0758	0.0515	0.2127	
	0.2	*	0.2111	0.1671	-0.0818	-0.0156	-0.0536	0.3825	-0.2229	-0.0014	0.0164	0.4883	-0.1390	-0.2040	0.0793	0.0491	0.2085
	0.4	*	0.2303	0.1366	-0.1633	-0.0327	-0.0937	0.3555	-0.4040	-0.0012	0.0383	0.7596	-0.1345	-0.1160	0.0897	0.0412	0.1966
	0.6	*	0.2588	0.0893	-0.2386	-0.0511	-0.1094	0.3269	-0.5142	0.0008	0.0640	1.1713	-0.1289	-0.0075	0.1059	0.0269	0.1795
	0.8	*	0.2909	0.0340	-0.2951	-0.0684	-0.0978	0.3099	-0.5476	0.0021	0.0859	1.6612	-0.1258	0.0878	0.1250	0.0082	0.1645
	1.0	*	0.3208	-0.0168	-0.3209	-0.0806	-0.0669	0.3085	-0.5206	-0.0013	0.0994	2.1587	-0.1281	0.1529	0.1420	-0.0078	0.1570
0.6	0.0	*	0.2995	0.1803	0.0	0.0	0.2918	0.0	0.0	0.0	0.4414	-0.1792	-0.2216	0.1131	0.0535	0.1637	
	0.2	*	0.3069	0.1682	-0.1033	-0.0214	-0.0535	0.2837	-0.2071	-0.0002	0.0178	0.5483	-0.1766	-0.1919	0.1173	0.0502	0.1594
	0.4	*	0.3275	0.1336	-0.2011	-0.0436	-0.0918	0.2644	-0.3725	0.0009	0.0379	0.8507	-0.1698	-0.1155	0.1296	0.0399	0.1480
	0.6	*	0.3567	0.0828	-0.2831	-0.0652	-0.1045	0.2453	-0.4706	0.0030	0.0578	1.2957	-0.1622	-0.0223	0.1477	0.0223	0.1341
	0.8	*	0.3881	0.0270	-0.3364	-0.0831	-0.0914	0.2352	-0.4996	0.0028	0.0728	1.8084	-0.1582	0.0604	0.1671	0.0031	0.1244
	1.0	*	0.4168	-0.0213	-0.3528	-0.0939	-0.0618	0.2358	-0.4761	-0.0029	0.0818	2.3167	-0.1593	0.1201	0.1833	-0.0115	0.1213
0.7	0.0	*	0.4173	0.1696	0.0	0.0	0.1982	0.0	0.0	0.0	0.5145	-0.2271	-0.2037	0.1618	0.0525	0.1174	
	0.2	*	0.4245	0.1569	-0.1300	-0.0272	-0.0489	0.1935	-0.1900	0.0018	0.0177	0.6431	-0.2237	-0.1789	0.1665	0.0482	0.1133
	0.4	*	0.4439	0.1215	-0.2471	-0.0535	-0.0826	0.1827	-0.3410	0.0042	0.0345	1.0018	-0.2151	-0.1155	0.1794	0.0357	0.1033
	0.6	*	0.4703	0.0720	-0.3370	-0.0768	-0.0925	0.1727	-0.4309	0.0059	0.0482	1.5184	-0.2063	-0.0374	0.1971	0.0170	0.0932
	0.8	*	0.4979	0.0204	-0.3872	-0.0938	-0.0800	0.1680	-0.4600	0.0038	0.0571	2.1018	-0.2016	0.0349	0.2149	-0.0022	0.0880
	1.0	*	0.5234	-0.0225	-0.3946	-0.1024	-0.0543	0.1695	-0.4425	-0.0039	0.0623	2.6738	-0.2010	0.0914	0.2290	-0.0145	0.0879
0.8	0.0	*	0.5628	0.1423	0.0	0.0	0.1146	0.0	0.0	0.0	0.6421	-0.2927	-0.1817	0.2269	0.0473	0.0737	
	0.2	*	0.5684	0.1307	-0.1663	-0.0312	-0.0390	0.1131	-0.1698	0.0044	0.0153	0.8130	-0.2890	-0.1635	0.2312	0.0424	0.0704
	0.4	*	0.5832	0.0994	-0.3097	-0.0598	-0.0654	0.1100	-0.3064	0.0081	0.0278	1.2856	-0.2803	-0.1159	0.2426	0.0287	0.0630
	0.6	*	0.6030	0.0571	-0.4109	-0.0826	-0.0727	0.1075	-0.3916	0.0091	0.0356	1.9591	-0.2722	-0.0543	0.2572	0.0102	0.0568
	0.8	*	0.6237	0.0146	-0.4594	-0.0973	-0.0631	0.1069	-0.4252	0.0052	0.0395	2.7158	-0.2680	0.0082	0.2709	-0.0066	0.0549
	1.0	*	0.6436	-0.0197	-0.4577	-0.1032	-0.0437	0.1085	-0.4169	-0.0038	0.0418	3.4593	-0.2651	0.0628	0.2816	-0.0159	0.0563
0.9	0.0	*	0.7457	0.0926	0.0	0.0	0.0444	0.0	0.0	0.0	0.9359	-0.3997	-0.1492	0.3201	0.0352	0.0330	
	0.2	*	0.7482	0.0848	-0.2256	-0.0299	-0.0230	0.0452	-0.1405	0.0068	0.0099	1.2198	-0.3986	-0.1406	0.3226	0.0306	0.0313
	0.4	*	0.7548	0.0641	-0.4133	-0.0557	-0.0385	0.0471	-0.2597	0.0113	0.0168	2.0087	-0.3972	-0.1155	0.3286	0.0183	0.0277
	0.6	*	0.7638	0.0368	-0.5371	-0.0745	-0.0437	0.0492	-0.3440	0.0115	0.0198	3.1469	-0.3980	-0.0760	0.3361	0.0034	0.0252
	0.8	*	0.7741	0.0097	-0.5891	-0.0852	-0.0391	0.0508	-0.3891	0.0066	0.0206	4.4537	-0.3992	-0.0260	0.3426	-0.0086	0.0251
	1.0	*	0.7856	-0.0121	-0.5785	-0.0884	-0.0285	0.0519	-0.3969	-0.0020	0.0209	5.7743	-0.3943	0.0268	0.3483	-0.0143	0.0265

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO NORMAL DISTRIBUTIONS

r = 0,8

003334

P	D	J(1,1)	J(1,2)	J(1,3)	J(1,4)	J(1,5)	J(2,2)	J(2,3)	J(2,4)	J(2,5)	J(3,3)	J(3,4)	J(3,5)	J(4,4)	J(4,5)	J(5,5)	

0.1	0.0	*	0.0080	0.0736	0.0	0.0	0.0	0.8411	0.0	0.0	0.0743	-0.0145	-0.1695	0.0031	0.0298	0.4282	
	0.2	*	0.0091	0.0719	-0.0249	-0.0015	-0.0253	0.8314	-0.3320	0.0008	0.0167	0.1825	-0.0133	-0.1127	0.0037	0.0291	0.4232
	0.4	*	0.0129	0.0658	-0.0605	-0.0039	-0.0469	0.8067	-0.6320	0.0026	0.0403	0.5454	-0.0084	0.0430	0.0057	0.0261	0.4061
	0.6	*	0.0206	0.0521	-0.1180	-0.0083	-0.0586	0.7755	-0.8569	0.0069	0.0709	1.2632	0.0022	0.2483	0.0101	0.0183	0.3791
	0.8	*	0.0319	0.0292	-0.1969	-0.0142	-0.0539	0.7526	-0.9635	0.0131	0.0982	2.4075	0.0131	0.4273	0.0164	0.0042	0.3531
1.0	*	0.0427	0.0021	-0.2717	-0.0187	-0.0353	0.7458	-0.9475	0.0161	0.1138	3.8758	0.0073	0.5312	0.0218	-0.0097	0.3378	
0.2	0.0	*	0.0324	0.1341	0.0	0.0	0.0	0.6923	0.0	0.0	0.0775	-0.0300	-0.1586	0.0129	0.0549	0.3605	
	0.2	*	0.0360	0.1283	-0.0501	-0.0057	-0.0441	0.6794	-0.3068	0.0030	0.0292	0.1893	-0.0275	-0.1092	0.0148	0.0524	0.3520
	0.4	*	0.0475	0.1090	-0.1134	-0.0137	-0.0771	0.6483	-0.5630	0.0087	0.0638	0.5402	-0.0197	0.0171	0.0213	0.0428	0.3283
	0.6	*	0.0662	0.0738	-0.1933	-0.0246	-0.0869	0.6169	-0.7221	0.0180	0.0976	1.1445	-0.0096	0.1636	0.0321	0.0230	0.3003
	0.8	*	0.0865	0.0286	-0.2714	-0.0344	-0.0713	0.6011	-0.7686	0.0252	0.1205	1.9452	-0.0102	0.2772	0.0431	-0.0016	0.2820
1.0	*	0.1024	-0.0135	-0.3201	-0.0397	-0.0416	0.6028	-0.7268	0.0231	0.1310	2.8160	-0.0304	0.3416	0.0507	-0.0184	0.2746	
0.3	0.0	*	0.0740	0.1808	0.0	0.0	0.0	0.5554	0.0	0.0	0.0813	-0.0466	-0.1471	0.0297	0.0752	0.2963	
	0.2	*	0.0806	0.1697	-0.0765	-0.0122	-0.0569	0.5430	-0.2819	0.0065	0.0376	0.1981	-0.0431	-0.1055	0.0335	0.0700	0.2864
	0.4	*	0.0998	0.1358	-0.1623	-0.0271	-0.0946	0.5157	-0.5038	0.0166	0.0752	0.5453	-0.0339	-0.0039	0.0449	0.0523	0.2624
	0.6	*	0.1265	0.0821	-0.2515	-0.0432	-0.1003	0.4930	-0.6268	0.0284	0.1045	1.0883	-0.0274	0.1071	0.0606	0.0224	0.2405
	0.8	*	0.1512	0.0230	-0.3195	-0.0547	-0.0787	0.4853	-0.6531	0.0327	0.1211	1.7381	-0.0367	0.1936	0.0738	-0.0073	0.2303
1.0	*	0.1696	-0.0265	-0.3483	-0.0600	-0.0444	0.4917	-0.6109	0.0247	0.1285	2.3941	-0.0619	0.2473	0.0822	-0.0240	0.2280	
0.4	0.0	*	0.1339	0.2129	0.0	0.0	0.0	0.4297	0.0	0.0	0.0859	-0.0647	-0.1349	0.0545	0.0903	0.2364	
	0.2	*	0.1431	0.1966	-0.1042	-0.0204	-0.0634	0.4207	-0.2566	0.0111	0.0421	0.2095	-0.0604	-0.1013	0.0601	0.0820	0.2266
	0.4	*	0.1677	0.1498	-0.2101	-0.0424	-0.1020	0.4027	-0.4512	0.0251	0.0780	0.5614	-0.0513	-0.0215	0.0755	0.0564	0.2059
	0.6	*	0.1981	0.0835	-0.3031	-0.0621	-0.1047	0.3906	-0.5539	0.0369	0.1007	1.0758	-0.0497	0.0654	0.0937	0.0195	0.1913
	0.8	*	0.2241	0.0169	-0.3605	-0.0740	-0.0810	0.3892	-0.5742	0.0368	0.1121	1.6555	-0.0651	0.1377	0.1072	-0.0121	0.1871
1.0	*	0.2437	-0.0354	-0.3738	-0.0789	-0.0458	0.3976	-0.5374	0.0237	0.1173	2.2184	-0.0918	0.1879	0.1156	-0.0276	0.1881	
0.5	0.0	*	0.2133	0.2293	0.0	0.0	0.0	0.3165	0.0	0.0	0.0915	-0.0847	-0.1216	0.0882	0.0997	0.1807	
	0.2	*	0.2239	0.2089	-0.1342	-0.0295	-0.0641	0.3123	-0.2305	0.0164	0.0427	0.2246	-0.0803	-0.0964	0.0952	0.0883	0.1724
	0.4	*	0.2504	0.1534	-0.2590	-0.0580	-0.1009	0.3052	-0.4028	0.0333	0.0742	0.5912	-0.0731	-0.0366	0.1127	0.0561	0.1571
	0.6	*	0.2805	0.0807	-0.3539	-0.0798	-0.1025	0.3028	-0.4943	0.0431	0.0906	1.1032	-0.0770	0.0323	0.1309	0.0154	0.1490
	0.8	*	0.3053	0.0120	-0.4018	-0.0915	-0.0796	0.3059	-0.5155	0.0384	0.0980	1.6609	-0.0970	0.0958	0.1433	-0.0159	0.1488
1.0	*	0.3252	-0.0402	-0.4026	-0.0957	-0.0461	0.3146	-0.4864	0.0212	0.1016	2.1923	-0.1237	0.1448	0.1511	-0.0297	0.1517	
0.6	0.0	*	0.3141	0.2287	0.0	0.0	0.0	0.2170	0.0	0.0	0.0987	-0.1075	-0.1069	0.1325	0.1026	0.1297	
	0.2	*	0.3242	0.2062	-0.1675	-0.0384	-0.0590	0.2176	-0.2027	0.0218	0.0395	0.2453	-0.1040	-0.0906	0.1399	0.0886	0.1239
	0.4	*	0.3483	0.1478	-0.3118	-0.0720	-0.0923	0.2206	-0.3562	0.0403	0.0653	0.6399	-0.1010	-0.0495	0.1567	0.0523	0.1146
	0.6	*	0.3742	0.0752	-0.4095	-0.0947	-0.0944	0.2258	-0.4427	0.0470	0.0766	1.1776	-0.1117	0.0042	0.1724	0.0110	0.1116
	0.8	*	0.3959	0.0089	-0.4495	-0.1059	-0.0749	0.2318	-0.4689	0.0381	0.0809	1.7550	-0.1357	0.0610	0.1826	-0.0185	0.1138
1.0	*	0.4151	-0.0407	-0.4394	-0.1093	-0.0450	0.2398	-0.4489	0.0181	0.0832	2.3026	-0.1615	0.1101	0.1893	-0.0305	0.1176	
0.7	0.0	*	0.4384	0.2093	0.0	0.0	0.0	0.1326	0.0	0.0	0.1083	-0.1341	-0.0901	0.1896	0.0976	0.0841	
	0.2	*	0.4460	0.1876	-0.2065	-0.0449	-0.0486	0.1370	-0.1719	0.0264	0.0326	0.2748	-0.1335	-0.0830	0.1958	0.0824	0.0813
	0.4	*	0.4634	0.1331	-0.3737	-0.0823	-0.0768	0.1471	-0.3084	0.0449	0.0524	0.7194	-0.1389	-0.0605	0.2086	0.0456	0.0778
	0.6	*	0.4817	0.0674	-0.4760	-0.1049	-0.0807	0.1572	-0.3942	0.0482	0.0598	1.3232	-0.1588	-0.0212	0.2193	0.0068	0.0781
	0.8	*	0.4984	0.0079	-0.5105	-0.1154	-0.0664	0.1648	-0.4291	0.0360	0.0620	1.9751	-0.1874	0.0294	0.2261	-0.0196	0.0813
1.0	*	0.5156	-0.0367	-0.4907	-0.1181	-0.0421	0.1715	-0.4203	0.0148	0.0633	2.5984	-0.2120	0.0788	0.2312	-0.0297	0.0853	
0.8	0.0	*	0.5898	0.1682	0.0	0.0	0.0	0.0655	0.0	0.0	0.1219	-0.1667	-0.0698	0.2633	0.0827	0.0451	
	0.2	*	0.5928	0.1512	-0.2545	-0.0479	-0.0333	0.0713	-0.1358	0.0285	0.0225	0.3208	-0.1731	-0.0721	0.2662	0.0687	0.0452
	0.4	*	0.5996	0.1085	-0.4528	-0.0846	-0.0550	0.0840	-0.2549	0.0454	0.0361	0.8594	-0.1949	-0.0689	0.2713	0.0363	0.0461
	0.6	*	0.6067	0.0566	-0.5670	-0.1065	-0.0612	0.0958	-0.3432	0.0456	0.0411	1.6102	-0.2301	-0.0456	0.2750	0.0035	0.0481
	0.8	*	0.6174	0.0087	-0.6001	-0.1165	-0.0534	0.1035	-0.3910	0.0322	0.0421	2.4458	-0.2662	-0.0029	0.2767	-0.0185	0.0511
1.0	*	0.6308	-0.0281	-0.5710	-0.1187	-0.0365	0.1088	-0.3972	0.0119	0.0424	3.2664	-0.2901	0.0464	0.2796	-0.0269	0.0545	
0.9	0.0	*	0.7734	0.1013	0.0	0.0	0.0	0.0190	0.0	0.0	0.1441	-0.2098	-0.0434	0.3607	0.0538	0.0149	
	0.2	*	0.7713	0.0927	-0.3203	-0.0396	-0.0149	0.0230	-0.0884	0.0241	0.0102	0.4071	-0.2335	-0.0540	0.3585	0.0448	0.0165
	0.4	*	0.7666	0.0700	-0.5708	-0.0703	-0.0274	0.0322	-0.1839	0.0374	0.0174	1.1645	-0.2935	-0.0709	0.3528	0.0240	0.0194
	0.6	*	0.7623	0.0401	-0.7175	-0.0898	-0.0344	0.0413	-0.2754	0.0367	0.0208	2.3177	-0.3642	-0.0696	0.3472	0.0020	0.0217
	0.8	*	0.7643	0.0103	-0.7626	-0.0997	-0.0336	0.0476	-0.3429	0.0256	0.0215	3.7112	-0.4207	-0.0418	0.3428	-0.0136	0.0234
1.0	*	0.7712	-0.0142	-0.7281	-0.1023	-0.0258	0.0512	-0.3736	0.0096	0.0213	5.1725	-0.4485	0.0040	0.3422	-0.0201	0.0254	

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO NORMAL DISTRIBUTIONS

r = 1.0

P	D	J(1,1)	J(1,2)	J(1,3)	J(1,4)	J(1,5)	J(2,2)	J(2,3)	J(2,4)	J(2,5)	J(3,3)	J(3,4)	J(3,5)	J(4,4)	J(4,5)	J(5,5)
0.1	0.0 *	0.0100	0.0900	0.0	0.0	0.0	0.8100	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.0050	0.0450	0.4050
0.1	0.2 *	0.0125	0.0859	-0.0441	-0.0041	-0.0306	0.8015	-0.3501	0.0043	0.0327	0.1654	0.0066	0.0546	0.0068	0.0415	0.3959
0.1	0.4 *	0.0204	0.0718	-0.1089	-0.0101	-0.0505	0.7821	-0.6399	0.0119	0.0639	0.7137	0.0256	0.1869	0.0121	0.0291	0.3751
0.1	0.6 *	0.0317	0.0472	-0.1960	-0.0164	-0.0529	0.7642	-0.8205	0.0211	0.0887	1.7020	0.0439	0.3310	0.0179	0.0096	0.3555
0.1	0.8 *	0.0419	0.0177	-0.2807	-0.0200	-0.0406	0.7556	-0.8802	0.0258	0.1039	3.0509	0.0401	0.4397	0.0219	-0.0086	0.3430
0.1	1.0 *	0.0488	-0.0092	-0.3357	-0.0212	-0.0208	0.7577	-0.8419	0.0230	0.1102	4.5698	0.0099	0.4964	0.0245	-0.0186	0.3376
0.2	0.0 *	0.0400	0.1600	0.0	0.0	0.0	0.6400	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.0200	0.0800	0.3200
0.2	0.2 *	0.0469	0.1477	-0.0838	-0.0133	-0.0468	0.6325	-0.3061	0.0139	0.0492	0.1601	0.0089	0.0356	0.0248	0.0698	0.3112
0.2	0.4 *	0.0645	0.1125	-0.1805	-0.0271	-0.0721	0.6186	-0.5389	0.0305	0.0871	0.6317	0.0259	0.1175	0.0351	0.0419	0.2944
0.2	0.6 *	0.0837	0.0635	-0.2753	-0.0368	-0.0710	0.6088	-0.6654	0.0423	0.1105	1.3516	0.0285	0.2057	0.0437	0.0088	0.2823
0.2	0.8 *	0.0982	0.0142	-0.3423	-0.0410	-0.0518	0.6080	-0.6943	0.0425	0.1225	2.2015	0.0057	0.2760	0.0493	-0.0161	0.2767
0.2	1.0 *	0.1076	-0.0254	-0.3682	-0.0423	-0.0248	0.6168	-0.6518	0.0314	0.1264	3.0629	-0.0338	0.3176	0.0534	-0.0271	0.2758
0.3	0.0 *	0.0900	0.2100	0.0	0.0	0.0	0.4900	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.0450	0.1050	0.2450
0.3	0.2 *	0.1003	0.1896	-0.1209	-0.0248	-0.0531	0.4879	-0.2660	0.0255	0.0549	0.1566	0.0080	0.0210	0.0516	0.0883	0.2402
0.3	0.4 *	0.1232	0.1367	-0.2385	-0.0452	-0.0802	0.4855	-0.4628	0.0481	0.0924	0.5873	0.0174	0.0723	0.0639	0.0488	0.2322
0.3	0.6 *	0.1454	0.0709	-0.3335	-0.0567	-0.0786	0.4862	-0.5672	0.0584	0.1124	1.1924	0.0066	0.1343	0.0732	0.0077	0.2277
0.3	0.8 *	0.1612	0.0097	-0.3845	-0.0613	-0.0576	0.4920	-0.5902	0.0525	0.1217	1.8615	-0.0260	0.1907	0.0794	-0.0205	0.2271
0.3	1.0 *	0.1724	-0.0369	-0.3918	-0.0628	-0.0280	0.5041	-0.5543	0.0345	0.1246	2.5097	-0.0674	0.2300	0.0843	-0.0316	0.2290
0.4	0.0 *	0.1600	0.2400	0.0	0.0	0.0	0.3600	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.0800	0.1200	0.1800
0.4	0.2 *	0.1711	0.2139	-0.1568	-0.0364	-0.0522	0.3641	-0.2284	0.0368	0.0532	0.1546	0.0046	0.0098	0.0865	0.0989	0.1801
0.4	0.4 *	0.1942	0.1498	-0.2917	-0.0622	-0.0800	0.3734	-0.3999	0.0629	0.0877	0.5644	0.0035	0.0398	0.0976	0.0524	0.1806
0.4	0.6 *	0.2153	0.0742	-0.3836	-0.0753	-0.0801	0.3833	-0.4948	0.0698	0.1048	1.1169	-0.0188	0.0856	0.1061	0.0070	0.1824
0.4	0.8 *	0.2307	0.0066	-0.4238	-0.0804	-0.0603	0.3938	-0.5199	0.0582	0.1121	1.7091	-0.0578	0.1347	0.1120	-0.0228	0.1849
0.4	1.0 *	0.2433	-0.0436	-0.4174	-0.0822	-0.0309	0.4072	-0.4927	0.0347	0.1145	2.2706	-0.0990	0.1742	0.1170	-0.0339	0.1885
0.5	0.0 *	0.2500	0.2500	0.0	0.0	0.0	0.2500	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.1250	0.1250	0.1250
0.5	0.2 *	0.2588	0.2218	-0.1923	-0.0464	-0.0462	0.2588	-0.1923	0.0462	0.0464	0.1539	-0.0014	0.0014	0.1292	0.1023	0.1292
0.5	0.4 *	0.2772	0.1540	-0.3443	-0.0769	-0.0738	0.2772	-0.3443	0.0738	0.0769	0.5574	-0.0153	0.0153	0.1362	0.0535	0.1362
0.5	0.6 *	0.2940	0.0752	-0.4359	-0.0917	-0.0771	0.2940	-0.4359	0.0771	0.0917	1.0942	-0.0488	0.0488	0.1423	0.0068	0.1423
0.5	0.8 *	0.3077	0.0055	-0.4671	-0.0977	-0.0606	0.3077	-0.4671	0.0606	0.0977	1.6643	-0.0927	0.0927	0.1469	-0.0235	0.1469
0.5	1.0 *	0.3211	-0.0459	-0.4496	-0.0998	-0.0333	0.3211	-0.4496	0.0333	0.0998	2.2016	-0.1330	0.1330	0.1516	-0.0346	0.1516
0.6	0.0 *	0.3600	0.2400	0.0	0.0	0.0	0.1600	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.1800	0.1200	0.0800
0.6	0.2 *	0.3641	0.2139	-0.2284	-0.0532	-0.0368	0.1711	-0.1568	0.0522	0.0364	0.1546	-0.0098	-0.0046	0.1801	0.0989	0.0865
0.6	0.4 *	0.3734	0.1498	-0.3999	-0.0877	-0.0629	0.1942	-0.2917	0.0800	0.0622	0.5644	-0.0398	-0.0035	0.1806	0.0524	0.0976
0.6	0.6 *	0.3833	0.0742	-0.4948	-0.1048	-0.0698	0.2153	-0.3836	0.0801	0.0753	1.1169	-0.0856	0.0188	0.1824	0.0070	0.1061
0.6	0.8 *	0.3938	0.0066	-0.5199	-0.1121	-0.0582	0.2307	-0.4238	0.0603	0.0804	1.7091	-0.1347	0.0578	0.1849	-0.0228	0.1120
0.6	1.0 *	0.4072	-0.0436	-0.4927	-0.1145	-0.0347	0.2433	-0.4174	0.0309	0.0822	2.2706	-0.1742	0.0990	0.1885	-0.0339	0.1170
0.7	0.0 *	0.4900	0.2100	0.0	0.0	0.0	0.0900	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.2450	0.1050	0.0450
0.7	0.2 *	0.4879	0.1896	-0.2660	-0.0549	-0.0255	0.1003	-0.1209	0.0531	0.0248	0.1566	-0.0210	-0.0080	0.2402	0.0883	0.0516
0.7	0.4 *	0.4855	0.1367	-0.4628	-0.0924	-0.0481	0.1232	-0.2385	0.0802	0.0452	0.5873	-0.0723	-0.0174	0.2322	0.0488	0.0639
0.7	0.6 *	0.4862	0.0709	-0.5672	-0.1124	-0.0584	0.1454	-0.3335	0.0786	0.0567	1.1924	-0.1343	-0.0066	0.2277	0.0077	0.0732
0.7	0.8 *	0.4920	0.0097	-0.5902	-0.1217	-0.0525	0.1612	-0.3845	0.0576	0.0613	1.8615	-0.1907	0.0260	0.2271	-0.0205	0.0794
0.7	1.0 *	0.5041	-0.0369	-0.5543	-0.1246	-0.0345	0.1724	-0.3918	0.0280	0.0628	2.5097	-0.2300	0.0674	0.2290	-0.0316	0.0843
0.8	0.0 *	0.6400	0.1600	0.0	0.0	0.0	0.0400	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.3200	0.0800	0.0200
0.8	0.2 *	0.6325	0.1477	-0.3061	-0.0492	-0.0139	0.0469	-0.0838	0.0468	0.0133	0.1601	-0.0356	-0.0089	0.3112	0.0698	0.0248
0.8	0.4 *	0.6186	0.1125	-0.5389	-0.0871	-0.0305	0.0645	-0.1805	0.0721	0.0271	0.6317	-0.1175	-0.0259	0.2944	0.0419	0.0351
0.8	0.6 *	0.6088	0.0635	-0.6654	-0.1105	-0.0423	0.0837	-0.2753	0.0710	0.0368	1.3516	-0.2057	-0.0285	0.2823	0.0088	0.0437
0.8	0.8 *	0.6080	0.0142	-0.6943	-0.1225	-0.0425	0.0982	-0.3423	0.0518	0.0410	2.2015	-0.2760	-0.0057	0.2767	-0.0161	0.0493
0.8	1.0 *	0.6168	-0.0254	-0.6518	-0.1264	-0.0314	0.1076	-0.3682	0.0248	0.0423	3.0629	-0.3176	0.0338	0.2758	-0.0271	0.0534
0.9	0.0 *	0.8100	0.0900	0.0	0.0	0.0	0.0100	0.0	0.0	0.0	0.0	0.0000	-0.0000	0.4050	0.0450	0.0050
0.9	0.2 *	0.8015	0.0859	-0.3501	-0.0327	-0.0043	0.0125	-0.0441	0.0306	0.0041	0.1654	-0.0546	-0.0066	0.3959	0.0415	0.0068
0.9	0.4 *	0.7821	0.0718	-0.6399	-0.0639	-0.0119	0.0204	-0.1089	0.0505	0.0101	0.7137	-0.1869	-0.0256	0.3751	0.0291	0.0121
0.9	0.6 *	0.7642	0.0472	-0.8205	-0.0887	-0.0211	0.0317	-0.1960	0.0529	0.0164	1.7020	-0.3310	-0.0439	0.3555	0.0096	0.0179
0.9	0.8 *	0.7556	0.0177	-0.8802	-0.1039	-0.0258	0.0419	-0.2807	0.0406	0.0200	3.0509	-0.4397	-0.0401	0.3430	-0.0086	0.0219
0.9	1.0 *	0.7577	-0.0092	-0.8419	-0.1102	-0.0230	0.0488	-0.3357	0.0208	0.0212	4.5698	-0.4964	-0.0099	0.3376	-0.0186	0.0245

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