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INFORMATION MATRIX FOR A MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

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ABSTRACT

This paper presents some numerical methods for computation of Fisher information matrix about the parameters of a mixture of two negative exponential distributions. It is shown that the computation of the information matrix leads to the computation of a particular kinds of integrals. An alternating power series is developed for the computation of these integrals, and the results are compared with those obtained by the Laguerre-Gauss quadrature and Romberg's algorithem. Some approximate results are also given for the case that the mixed distributions are not well separated.

A brief table is provided which may be useful in practice.

1. INTRODUCTION

Consider

$$f(x) = pf_1(x) + qf_2(x), \tag{1.1}$$

where, for $j = 1,2$,

$$f_j(x) = \alpha_j \exp(-\alpha_j x) \tag{1.2}$$

for $x \geq 0$ and zero elsewhere, with $\alpha_j > 0$, $0 < p < 1$, and $q = 1-p$. The function $f(x)$ is the probability density function of a mixture of two negative exponential distributions with mixing proportions p and q. This distribution is of particular interest in the analysis of experimental data regarding life testing and electronic equipment failure times, and the point estimation of its parameters has already received some attention in the literature [2,3,6,9-12].

The purpose of this paper is to present some numerical methods for computation of the information matrix about the three parameters α_1, α_2, p of the density (1), which can be extended to a finite mixture of more than two exponential densities. The information matrix that we shall use is that of Fisher, namely the symmetric positive definite $\ell \times \ell$ matrix $I(\theta) = || I(\theta_s, \theta_t) ||$ with

$$I(\theta_s, \theta_t) = E\left[\frac{\partial \log f(x,\theta)}{\partial \theta_s} \frac{\partial \log f(x,\theta)}{\partial \theta_t}\right], \quad s, t=1,2,\ldots,\ell \tag{1.3},$$

where $f(x,\theta)$ is the probability density function of an arbitrary random variable X depending on an ℓ-dimensional parameter $\theta = (\theta_1, \theta_2, \ldots, \theta_\ell)$. The expectation in (1.3) is taken with respect to the density $f(x,\theta)$, and it is assumed that all the derivatives and expectations in question exist.

The Fisher information, whose properties are known, is a natural and convenient measure of the expected precision in both classical and Bayesian Statistics [5].

The first work related to this paper is given by Hill [5], who computes the Fisher information about the proportion p, when α_1 and α_2 are known, by a power series. The whole information matrix for the parameters of (1.1) is computed by the author [1] using the Laguerre-Gauss quadrature numerical integration. Here we will show that the computation of the information matrix leads to the computation of a particular kind of integrals. These integrals are computed by a power series, and the numerical results are compared with those obtained by the Laguerre-Gauss quadrature, Romberg's algorithm, and Taylor series. A brief table is provided which may be useful in practice for finding the information matrix.

2. CALCULATION OF THE INFORMATION MATRIX

Simple calculation shows that for the density (1.1) the following relations hold:

$$\begin{aligned} &\partial \log f(x)/\partial \alpha_i = p_i(1/\alpha_i - x) f_i(x)/f(x), \quad i=1,2 \\ &p\, \partial \log f(x)/\partial p = 1 - f_2(x)/f(x), \end{aligned} \tag{2.1}$$

where $p_1 = p$ and $p_2 = q$. For $i, j = 1,2$ and $m,n = 0,1$, let

$$M_{mn}(f_i, f_j) = \int_0^\infty (1/\alpha_i - x)^m (1/\alpha_j - x)^n [f_i(x) f_j(x)/f(x)]dx. \tag{2.2}$$

It is easy to show that the improper integrals (2.2) exist. Now, using (1.3),(2.1), and (2.2), we obtain

$$I(\alpha_i, \alpha_j) = p_i p_j \, M_{11}(f_i, f_j) \qquad i \le j, \quad i,j = 1,2$$

$$I(\alpha_1, p) = -M_{10}(f_1, f_2)$$

$$I(\alpha_2, p) = M_{01}(f_1, f_2) \tag{2.3}$$

$$I(p,p) = [1 - M_{00}(f_1, f_2)]/pq.$$

Let $\alpha_1 < \alpha_2$, without loss of generality, and take $h = \alpha_1/\alpha_2$, where $0 < h < 1$. Now, consider the simple linear transformation $y = \alpha_2 x$. This transformation sends the density (1.1) to the density

$$g(y) = p \, g_1(y) + q \, g_2(y) \tag{2.4}$$

where, for $j = 1,2$,

$$g_j(y) = h_j \exp(-h_j y) \tag{2.5}$$

with the conventions $h_1 = h$ and $h_2 = 1$. Thus the mixture (1.1) with three parameters α_1, α_2, p is reduced to the mixture (2.4) with two parameters h and p which are between zero and one. Using this transformation the formula (2.2) becomes

$$M_{mn}(f_i, f_j) = G_{mn}(g_i, g_j)/\alpha_i^m \, \alpha_j^n \tag{2.6}$$

with

$$G_{mn}(g_i, g_j) = \int_0^\infty (1-h_i y)^m (1-h_j y)^n [g_i(y) g_j(y)/g(y)] dy \tag{2.7}$$

Now, from (2.3) and (2.6), we obtain the following items, which we call the scaled elements of the information matrix.

$$\alpha_i\alpha_j\, I(\alpha_i,\alpha_j) = p_i p_j\, G_{11}(g_i,g_j), \quad i \le j, \quad i,j = 1,2$$

$$\alpha_1 I(\alpha_1,p) = -\,G_{10}(g_1,g_2)$$

$$\alpha_2 I(\alpha_2,p) = G_{01}(g_1,g_2) \qquad (2.8)$$

$$I(p,p) = [1 - G_{00}(g_1,g_2)]/pq.$$

Denoting the information matrix, in the order α_1, α_2, p for the parameters, by $I = ||(s,t)||$ and the scaled information matrix, which depends only on h and p, by $J = ||J(s,t)||$, $s,t = 1,2,3$, and considering the diagonal matrix $\Delta = \text{diag.}(1/\alpha_1,\ 1/\alpha_2,\ 1)$, we have

$$I = \Delta\ J\ \Delta \qquad (2.9).$$

Hence to compute I, it is enough to compute J and then apply (2.9). But to compute J, we have to compute (2.7).

Letting

$$m_{ijk} = \int_0^\infty y^k[g_i(y)g_j(y)/g(y)]dy, \qquad (2.10)$$

for $i,j = 1,2$ and $k = 0,1,2$, from (2.7) we obtain

$$G_{11}(g_i,g_j) = m_{ij0} - (h_i+h_j)m_{ij1} + h_i h_j m_{ij2}$$

$$G_{10}(g_1,g_2) = m_{120} - h\ m_{121}$$

$$G_{01}(g_1,g_2) = m_{120} - m_{121} \qquad (2.11)$$

$$G_{00}(g_1,g_2) = m_{120}\ .$$

Thus, the computation of $G_{mn}(g_i,g_j)$ leads to the computation of the moment-type integrals (2.10).

3. COMPUTATION OF m_{ijk}

In this section we give a power series expansion for the computation of m_{ijk}, and we also suggest the application of the Laguerre-Gauss quadrature and Romberg's algorithm.

We observe, from (2.4) and (2.5), that $pg_1(y)/q\,g_2(y) < 1$ if y is in the interval $(0, a)$ and $q\,g_2(y)/p\,g_1(y) < 1$ if y is in the interval (a,∞), where

$$a = (\log q/ph)/(1-h) \tag{3.1}$$

is the positive root of $p\,q_1(y) = q\,g_2(y)$ if it exists. When $a \le 0$, i.e., when $p \ge 1/(1+h)$, we have $q\,g_2(y)/p\,g_1(y) < 1$ for all y in the interval $(0, \infty)$. Now, breaking the interval $(0, \infty)$ into intervals $(0, a)$ and (a, ∞), dividing the numerator and denominator of $g_i(y)g_j(y)/g(y)$ by $q\,g_2(y)$ or $p\,g_1(y)$ depending on whether y is in $(0, a)$ or in (a, ∞), and using geometric expansions, we obtain

$$m_{ij.k} = \sum_{N=0}^{\infty} \left[\int_0^a y^k A_{ijN}(y)\,dy + \int_a^{\infty} y^k B_{ijN}(y)\,dy\right], \tag{3.2}$$

where

$$\begin{aligned} A_{ijN}(y) &= (-1)^N p^N q^{-N-1} g_i(y)g_j(y)[g_1(y)]^N[g_2(y)]^{-N-1} \\ B_{ijN}(y) &= (-1)^N q^N p^{-N-1} g_i(y)g_j(y)[g_1(y)]^{-N-1}[g_2(y)]^N. \end{aligned} \tag{3.3}$$

From (2.4) - (2.5) and (3.2) - (3.3), after some calculation we obtain

$$m_{ijk} = \sum_{N=0}^{\infty}(-1)^N \ (a_{ijN}C_{ijkN} + b_{ijN}D_{ijkN}), \tag{3.4}$$

where

$$\begin{aligned} a_{ijN} &= p^N q^{-N-1} h^N h_i h_j \\ b_{ijN} &= p^{-N-1} q^N h^{-N-1} h_i h_j \end{aligned} \tag{3.5}$$

and

$$C_{ijkN} = \int_0^a y^k \exp(-c_{ijN}y)\,dy$$
$$D_{ijkN} = \int_a^\infty y^k \exp(-d_{ijN}y)\,dy \tag{3.6}$$

with

$$c_{ijN} = h_i + h_j - 1 - N + Nh$$
$$d_{ijN} = h_i + h_j - h + N - Nh \;. \tag{3.7}$$

For fixed i, j, N, let us denote c_{ijN}, d_{ijN}, C_{ijkN}, D_{ijkN} respectively by c, d, C_k, D_k. Now from (3.6) we obtain

$$C_0 = [1 - \exp(-ac)]/c$$
$$C_1 = (C_0 + acC_0 - a)/c$$
$$C_2 = 2C_1/c + a^2C_0 - a^2/c$$
$$D_0 = \exp(-ad)/d \tag{3.8}$$
$$D_1 = D_0(a + 1/d)$$
$$D_2 = D_0(a^2 + 2a/d + 2/d^2)$$

Thus, we can compute m_{ijk} by using the alternating series (3.4) and the relations (3.5), (3.7), and (3.8).

Next, we look at the Laguerre-Gauss quadrature formula. For a convergent integral of the form

$$I(G) = \int_0^\infty e^{-z}G(z)\,dz \tag{3.9}$$

this formula is

$$I(G) = \sum_{K=1}^{N} w_K G(z_K) + R_N(G), \tag{3.10}$$

where z_K and w_K, which are referred to respectively as nodes and weights can be determined through appropriate Laguerre polynomials, and they are already known and tabulated [4,7]. The summation $\sum_{K=0}^{N} w_k G(z_K)$ approximates $I(G)$, and the error $R_N(G)$ depends on N and the behavior of G and its derivatives on $(0, \infty)$. To have small error it is desirable that G and its derivatives remain bounded on $(0, \infty)$.

We now show that (2.10) can be put in the form (3.9). To guarantee the boundedness of $G(z)$ and its derivatives, by introducing an extra factor $\exp(-hy/2)$ and using the transformation $y = 2z/h$, we obtain

$$m_{ijk} = (2/h)^{k+1} h_i h_j \int_0^\infty \frac{\exp(-z)\,dz}{ph\,\exp(-H_{ij}z) + q\,\exp(-L_{ij}z)} \qquad (3.11),$$

where

$$\begin{aligned} H_{ij} &= (3h-2h_i-2h_j)/h \\ L_{ij} &= (2-2h_i-2h_j+h)/h. \end{aligned} \qquad (3.12).$$

Now, we can apply the quadrature formula to (3.11) with $G(z) = 1/[ph\,\exp(-H_{ij}z) + q\,\exp(-L_{ij}z)]$.

It is also possible to apply the Romberg's algorithm which is known [8], for the numerical evaluation of (3.11). This can be done by breaking the range of integration and using some appropriate transformations to change the integral (4.5) into sum of two integrals on the interval (0, 1).

Computation shows that the numerical results obtained by the above methods agree at least up to four decimal figures. However, the series (3.4) is more convenient for numerical work and we can also estimate the error easily.

4. CALCULATION OF m_{ijk} WHEN h IS CLOSE TO ONE

When the densities are not well separated, i.e., α_1 and α_2 are clost to each other or h is close to one, calculation shows that the Taylor approximation of $g_i(y)g_j(y)/g(j)$, as a function of h in the neighborhood of $h = 1$ is of the form $P_{ij}(y)\exp(-y)$, where $P_{ij}(y)$ is a polynomial whose coefficients depend on h and p. Using the first four terms of the Taylor series when $1 - h = t$ is close to zero, we obtain by simple calculation

$$\begin{aligned}
m_{110} &= 1 + q^2t^2 - 2q^2(1+p)t^3 + 0(t^3)\\
m_{111} &= 1 - (1+q)t + (3q^2+q+1)t^2 + (p-11pq^2-12q^2)t^3 + 0(t^3)\\
m_{112} &= 2-4(1+q)t+(6q+14q^2+6)t^2+(144p-68p^2-64pq^2-100)t^3+0(t^3)\\
m_{120} &= 1 - pqt^2 + 2pq^2t^3 + 0(t^3) \qquad (4.1)\\
m_{121} &= 1 - qt + (3q^2-2q)t^2+ (11pq^2-pq-q)t^3 + 0(t^3)\\
m_{122} &= 2-4qt + (7q^2-7pq-q)t^2 + (64pq^2-4pq-8q)t^3 + 0(t^3).\\
m_{220} &= 1 + p^2t^2 - 2p^2q\ t^3 + 0(t^3)\\
m_{221} &= 1 + pt + (3p^2-p)t^2 + (2p+p^2-11p^2q)t^3 + 0(t^3)\\
m_{222} &= 2 + 4pt + (p+7p^2-7pq)t^2 - (8p+3p^2-64p^2q)t^3 + 0(t^3)
\end{aligned}$$

Using (2.8), (2.11), and the above relations, we can easily find a good approximate value for the information matrix when α_1 and α_2 are close to each other.

We observe, from the relations (2.3), that when α_1/α_2 goes to one, that is when the mixed densities become closer and closer to each other, the information matrix I approaches to a singular matrix continuously, with some diagonal elements equal to zero. The same assertion is true when p goes to zero or one.

It is known that for a sample of size, say N, I^{-1}/N is the asymptotic covariance matrix of the maximum likelihood estimates of the parameters, and the diagonal elements of I^{-1}/N are the variances of the asymptotic distributions of these estimates. Combining this fact with the above assertion, we conclude that for estimating the parameters of a mixture of two exponential distributions, which are not well separated or with a mixing proportion close to zero, a huge sample is required and the estimation may be impractical.

We now look at a numerical example. Consider a mixture of two exponential distributions with $\alpha_1 = .18$, $\alpha_2 = .20$, $p = .7$. Here we have $h = .9$. We have computed the information matrix by different methods. They all agree at least up to four decimal figures and the result is the symmetric positive definite matrix

$$I = \begin{bmatrix} 16.0462 & 5.5111 & -0.4105 \\ & 1.9725 & -0.1440 \\ & & 0.0106 \end{bmatrix}$$

Finally, we have provided a short table for the standard elements of the information matrix, which may be useful in practice for finding the information about all or some of the parameters.

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SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

p	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)
0.05	0.05 *	0.0404	-0.0211	-0.2349	0.8252	-0.6067	15.1738	0.10 *	0.0380	-0.0193	-0.3258	0.7907	-0.8241	11.5028
	0.15 *	0.0373	-0.0112	-0.3559	0.7748	-0.9295	8.7859	0.20 *	0.0368	-0.0007	-0.3509	0.7685	-0.9728	6.6988
	0.25 *	0.0357	0.0107	-0.3254	0.7681	-0.9758	5.0723	0.30 *	0.0339	0.0219	-0.2883	0.7718	-0.9507	3.8025
	0.35 *	0.0312	0.0321	-0.2462	0.7784	-0.9054	2.8137	0.40 *	0.0278	0.0410	-0.2034	0.7873	-0.8455	2.0498
	0.45 *	0.0240	0.0483	-0.1629	0.7977	-0.7751	1.4667	0.50 *	0.0201	0.0538	-0.1266	0.8092	-0.6979	1.0285
	0.55 *	0.0163	0.0576	-0.0957	0.8213	-0.6168	0.7049	0.60 *	0.0128	0.0597	-0.0705	0.8335	-0.5343	0.4710
	0.65 *	0.0099	0.0604	-0.0507	0.8455	-0.4527	0.3055	0.70 *	0.0076	0.0599	-0.0357	0.8568	-0.3737	0.1911
	0.75 *	0.0058	0.0585	-0.0247	0.8673	-0.2986	0.1139	0.80 *	0.0046	0.0565	-0.0166	0.8766	-0.2285	0.0632
	0.85 *	0.0038	0.0543	-0.0107	0.8847	-0.1636	0.0311	0.90 *	0.0032	0.0520	-0.0062	0.8917	-0.1041	0.0123
	0.95 *	0.0028	0.0497	-0.0028	0.8976	-0.0497	0.0028	1.00 *	0.0025	0.0475	0.0	0.9025	0.0	0.0
0.10	0.05 *	0.0825	-0.0329	-0.2247	0.7257	-0.4873	8.1978	0.10 *	0.0773	-0.0306	-0.3175	0.6805	-0.6489	6.3908
	0.15 *	0.0757	-0.0194	-0.3559	0.6588	-0.7269	5.0337	0.20 *	0.0748	-0.0044	-0.3621	0.6483	-0.7606	3.9690
	0.25 *	0.0735	0.0121	-0.3481	0.6448	-0.7660	3.1169	0.30 *	0.0711	0.0288	-0.3214	0.6462	-0.7517	2.4293
	0.35 *	0.0675	0.0448	-0.2873	0.6511	-0.7230	1.8737	0.40 *	0.0627	0.0595	-0.2497	0.6589	-0.6833	1.4261
	0.45 *	0.0569	0.0725	-0.2113	0.6689	-0.6353	1.0682	0.50 *	0.0504	0.0834	-0.1742	0.6808	-0.5809	0.7850
	0.55 *	0.0437	0.0921	-0.1401	0.6941	-0.5219	0.5639	0.60 *	0.0370	0.0984	-0.1098	0.7084	-0.4599	0.3942
	0.65 *	0.0308	0.1024	-0.0839	0.7233	-0.3964	0.2667	0.70 *	0.0253	0.1043	-0.0624	0.7382	-0.3327	0.1731
	0.75 *	0.0208	0.1043	-0.0451	0.7529	-0.2700	0.1064	0.80 *	0.0172	0.1028	-0.0314	0.7668	-0.2094	0.0605
	0.85 *	0.0145	0.1003	-0.0207	0.7796	-0.1516	0.0303	0.90 *	0.0125	0.0971	-0.0122	0.7911	-0.0974	0.0121
	0.95 *	0.0111	0.0936	-0.0055	0.8013	-0.0468	0.0027	1.00 *	0.0100	0.0900	0.0	0.8100	0.0	0.0
0.15	0.05 *	0.1253	-0.0413	-0.2216	0.6449	-0.4260	5.8379	0.10 *	0.1176	-0.0384	-0.3154	0.5948	-0.5569	4.6088
	0.15 *	0.1150	-0.0250	-0.3580	0.5702	-0.6183	3.6837	0.20 *	0.1137	-0.0072	-0.3702	0.5574	-0.6440	2.9512
	0.25 *	0.1122	0.0126	-0.3626	0.5518	-0.6477	2.3584	0.30 *	0.1096	0.0329	-0.3420	0.5512	-0.6362	1.8732
	0.35 *	0.1056	0.0527	-0.3130	0.5542	-0.6136	1.4746	0.40 *	0.1001	0.0714	-0.2792	0.5602	-0.5826	1.1469
	0.45 *	0.0932	0.0884	-0.2432	0.5687	-0.5448	0.8789	0.50 *	0.0851	0.1033	-0.2069	0.5793	-0.5019	0.6614
	0.55 *	0.0764	0.1160	-0.1719	0.5917	-0.4548	0.4868	0.60 *	0.0673	0.1260	-0.1395	0.6056	-0.4046	0.3488
	0.65 *	0.0584	0.1334	-0.1102	0.6206	-0.3523	0.2417	0.70 *	0.0501	0.1381	-0.0847	0.6364	-0.2988	0.1605
	0.75 *	0.0427	0.1404	-0.0630	0.6524	-0.2451	0.1006	0.80 *	0.0365	0.1404	-0.0449	0.6684	-0.1920	0.0582
	0.85 *	0.0315	0.1387	-0.0302	0.6837	-0.1403	0.0296	0.90 *	0.0277	0.1357	-0.0181	0.6980	-0.0908	0.0120
	0.95 *	0.0247	0.1318	-0.0082	0.7109	-0.0440	0.0027	1.00 *	0.0225	0.1275	0.0	0.7225	0.0	0.0
0.20	0.05 *	0.1687	-0.0475	-0.2218	0.5748	-0.3866	4.6635	0.10 *	0.1588	-0.0438	-0.3167	0.5228	-0.4966	3.7058
	0.15 *	0.1551	-0.0287	-0.3622	0.4969	-0.5457	2.9850	0.20 *	0.1534	-0.0089	-0.3781	0.4828	-0.5650	2.4133
	0.25 *	0.1518	0.0131	-0.3747	0.4758	-0.5662	1.9482	0.30 *	0.1492	0.0357	-0.3582	0.4736	-0.5553	1.5648
	0.35 *	0.1451	0.0581	-0.3329	0.4749	-0.5356	1.2467	0.40 *	0.1392	0.0794	-0.3020	0.4792	-0.5092	0.9826
	0.45 *	0.1317	0.0993	-0.2679	0.4859	-0.4775	0.7636	0.50 *	0.1228	0.1173	-0.2326	0.4948	-0.4415	0.5832
	0.55 *	0.1127	0.1329	-0.1975	0.5057	-0.4021	0.4360	0.60 *	0.1019	0.1460	-0.1638	0.5182	-0.3598	0.3175
	0.65 *	0.0909	0.1564	-0.1324	0.5322	-0.3154	0.2235	0.70 *	0.0803	0.1639	-0.1041	0.5474	-0.2695	0.1508
	0.75 *	0.0704	0.1686	-0.0791	0.5634	-0.2228	0.0960	0.80 *	0.0617	0.1706	-0.0575	0.5798	-0.1759	0.0562
	0.85 *	0.0543	0.1703	-0.0392	0.5960	-0.1296	0.0289	0.90 *	0.0484	0.1681	-0.0238	0.6118	-0.0845	0.0118
	0.95 *	0.0437	0.1645	-0.0109	0.6265	-0.0412	0.0027	1.00 *	0.0400	0.1600	0.0	0.6400	0.0	0.0
0.25	0.05 *	0.2124	-0.0520	-0.2241	0.5120	-0.3586	3.9744	0.10 *	0.2007	-0.0473	-0.3204	0.4599	-0.4524	3.1666
	0.15 *	0.1960	-0.0310	-0.3679	0.4337	-0.4920	2.5616	0.20 *	0.1940	-0.0097	-0.3866	0.4191	-0.5057	2.0820
	0.25 *	0.1923	0.0138	-0.3861	0.4111	-0.5044	1.6912	0.30 *	0.1897	0.0380	-0.3725	0.4077	-0.4932	1.3679
	0.35 *	0.1857	0.0619	-0.3499	0.4076	-0.4750	1.0985	0.40 *	0.1798	0.0851	-0.3212	0.4103	-0.4514	0.8732
	0.45 *	0.1721	0.1069	-0.2887	0.4154	-0.4236	0.6850	0.50 *	0.1627	0.1269	-0.2542	0.4226	-0.3925	0.5285
	0.55 *	0.1518	0.1449	-0.2192	0.4317	-0.3584	0.3994	0.60 *	0.1400	0.1603	-0.1848	0.4425	-0.3219	0.2941
	0.65 *	0.1275	0.1731	-0.1520	0.4550	-0.2835	0.2095	0.70 *	0.1150	0.1830	-0.1215	0.4689	-0.2436	0.1429
	0.75 *	0.1031	0.1900	-0.0938	0.4840	-0.2025	0.0920	0.80 *	0.0921	0.1941	-0.0692	0.4999	-0.1610	0.0545
	0.85 *	0.0825	0.1955	-0.0477	0.5161	-0.1194	0.0283	0.90 *	0.0744	0.1945	-0.0293	0.5323	-0.0783	0.0117
	0.95 *	0.0678	0.1917	-0.0135	0.5479	-0.0384	0.0027	1.00 *	0.0625	0.1875	0.0	0.5625	0.0	0.0

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

p	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)
0.30	0.05	0.2569	-0.0549	-0.2287	0.4549	-0.3368	3.5311	0.10	0.2433	-0.0494	-0.3260	0.4038	-0.4179	2.8161
	0.15	0.2378	-0.0319	-0.3752	0.3781	-0.4494	2.2821	0.20	0.2354	-0.0097	-0.3959	0.3633	-0.4583	1.8600
	0.25	0.2337	0.0147	-0.3975	0.3548	-0.4546	1.5163	0.30	0.2313	0.0398	-0.3861	0.3505	-0.4427	1.2319
	0.35	0.2275	0.0647	-0.3655	0.3493	-0.4252	0.9943	0.40	0.2218	0.0889	-0.3384	0.3506	-0.4035	0.7949
	0.45	0.2142	0.1119	-0.3072	0.3542	-0.3785	0.6277	0.50	0.2046	0.1333	-0.2734	0.3597	-0.3508	0.4877
	0.55	0.1935	0.1528	-0.2384	0.3671	-0.3208	0.3715	0.60	0.1809	0.1700	-0.2035	0.3763	-0.2889	0.2758
	0.65	0.1675	0.1846	-0.1695	0.3871	-0.2552	0.1982	0.70	0.1538	0.1966	-0.1373	0.3994	-0.2201	0.1365
	0.75	0.1402	0.2056	-0.1074	0.4131	-0.1839	0.0887	0.80	0.1274	0.2116	-0.0802	0.4278	-0.1469	0.0530
	0.85	0.1157	0.2147	-0.0560	0.4433	-0.1096	0.0278	0.90	0.1056	0.2152	-0.0347	0.4592	-0.0723	0.0115
	0.95	0.0970	0.2135	-0.0161	0.4749	-0.0356	0.0027	1.00	0.0900	0.2100	0.0	0.4900	0.0	0.0
0.35	0.05	0.3018	-0.0569	-0.2338	0.4022	-0.3204	3.2396	0.10	0.2868	-0.0501	-0.3333	0.3532	-0.3897	2.5773
	0.15	0.2805	-0.0318	-0.3841	0.3283	-0.4140	2.0884	0.20	0.2779	-0.0090	-0.4062	0.3138	-0.4187	1.7037
	0.25	0.2762	0.0159	-0.4094	0.3053	-0.4126	1.3914	0.30	0.2740	0.0413	-0.3996	0.3001	-0.3999	1.1333
	0.35	0.2704	0.0666	-0.3804	0.2980	-0.3827	0.9176	0.40	0.2651	0.0912	-0.3545	0.2982	-0.3623	0.7365
	0.45	0.2578	0.1148	-0.3242	0.3005	-0.3394	0.5841	0.50	0.2484	0.1369	-0.2909	0.3046	-0.3144	0.4562
	0.55	0.2373	0.1573	-0.2560	0.3104	-0.2877	0.3494	0.60	0.2245	0.1757	-0.2206	0.3179	-0.2594	0.2611
	0.65	0.2106	0.1917	-0.1857	0.3270	-0.2296	0.1889	0.70	0.1960	0.2051	-0.1520	0.3376	-0.1986	0.1310
	0.75	0.1813	0.2158	-0.1202	0.3496	-0.1666	0.0857	0.80	0.1670	0.2236	-0.0907	0.3629	-0.1336	0.0516
	0.85	0.1536	0.2284	-0.0639	0.3772	-0.1002	0.0273	0.90	0.1417	0.2305	-0.0399	0.3922	-0.0665	0.0114
	0.95	0.1313	0.2300	-0.0187	0.4075	-0.0329	0.0027	1.00	0.1225	0.2275	0.0	0.4225	0.0	0.0
0.40	0.05	0.3473	-0.0575	-0.2410	0.3535	-0.3065	3.0423	0.10	0.3310	-0.0497	-0.3425	0.3071	-0.3657	2.4119
	0.15	0.3241	-0.0308	-0.3945	0.2835	-0.3836	1.9510	0.20	0.3213	-0.0076	-0.4177	0.2694	-0.3843	1.5908
	0.25	0.3197	0.0172	-0.4221	0.2606	-0.3760	1.2996	0.30	0.3178	0.0425	-0.4133	0.2552	-0.3624	1.0596
	0.35	0.3147	0.0677	-0.3950	0.2525	-0.3454	0.8595	0.40	0.3098	0.0923	-0.3700	0.2519	-0.3260	0.6915
	0.45	0.3029	0.1159	-0.3403	0.2531	-0.3047	0.5501	0.50	0.2940	0.1382	-0.3073	0.2559	-0.2820	0.4312
	0.55	0.2831	0.1590	-0.2723	0.2603	-0.2579	0.3317	0.60	0.2705	0.1779	-0.2365	0.2662	-0.2326	0.2490
	0.65	0.2565	0.1948	-0.2008	0.2737	-0.2062	0.1811	0.70	0.2415	0.2093	-0.1658	0.2826	-0.1788	0.1263
	0.75	0.2261	0.2213	-0.1323	0.2929	-0.1503	0.0832	0.80	0.2107	0.2305	-0.1007	0.3046	-0.1210	0.0504
	0.85	0.1961	0.2369	-0.0716	0.3174	-0.0911	0.0268	0.90	0.1825	0.2405	-0.0451	0.3312	-0.0607	0.0113
	0.95	0.1704	0.2414	-0.0212	0.3455	-0.0302	0.0027	1.00	0.1600	0.2400	0.0	0.3600	0.0	0.0
0.45	0.05	0.3908	-0.0573	-0.2499	0.3081	-0.2950	2.9149	0.10	0.3761	-0.0482	-0.3535	0.2648	-0.3446	2.2989
	0.15	0.3688	-0.0289	-0.4065	0.2427	-0.3565	1.8537	0.20	0.3659	-0.0058	-0.4306	0.2293	-0.3535	1.5086
	0.25	0.3645	0.0186	-0.4357	0.2206	-0.3431	1.2313	0.30	0.3629	0.0435	-0.4275	0.2151	-0.3287	1.0038
	0.35	0.3602	0.0682	-0.4098	0.2120	-0.3118	0.8146	0.40	0.3559	0.0922	-0.3853	0.2112	-0.2931	0.6562
	0.45	0.3496	0.1154	-0.3558	0.2114	-0.2733	0.5230	0.50	0.3413	0.1374	-0.3229	0.2127	-0.2524	0.4110
	0.55	0.3310	0.1581	-0.2878	0.2159	-0.2306	0.3171	0.60	0.3188	0.1771	-0.2516	0.2205	-0.2080	0.2388
	0.65	0.3050	0.1944	-0.2150	0.2264	-0.1845	0.1744	0.70	0.2901	0.2095	-0.1788	0.2337	-0.1602	0.1222
	0.75	0.2743	0.2224	-0.1438	0.2424	-0.1350	0.0809	0.80	0.2584	0.2327	-0.1104	0.2524	-0.1090	0.0493
	0.85	0.2427	0.2404	-0.0799	0.2636	-0.0823	0.0264	0.90	0.2280	0.2453	-0.0501	0.2759	-0.0551	0.0111
	0.95	0.2145	0.2476	-0.0238	0.2890	-0.0276	0.0026	1.00	0.2025	0.2475	0.0	0.3025	0.0	0.0
0.50	0.05	0.4401	-0.0560	-0.2606	0.2658	-0.2850	2.8426	0.10	0.4221	-0.0458	-0.3666	0.2260	-0.3254	2.2261
	0.15	0.4146	-0.0263	-0.4206	0.2056	-0.3317	1.7868	0.20	0.4117	-0.0036	-0.4451	0.1930	-0.3252	1.4494
	0.25	0.4099	0.0202	-0.4504	0.1847	-0.3130	1.1805	0.30	0.4093	0.0442	-0.4425	0.1792	-0.2977	0.9613
	0.35	0.4072	0.0679	-0.4250	0.1757	-0.2808	0.7798	0.40	0.4035	0.0908	-0.3997	0.1739	-0.2638	0.6317
	0.45	0.3980	0.1133	-0.3711	0.1735	-0.2444	0.5012	0.50	0.3904	0.1346	-0.3381	0.1748	-0.2252	0.3944
	0.55	0.3808	0.1547	-0.3027	0.1766	-0.2054	0.3049	0.60	0.3693	0.1735	-0.2659	0.1800	-0.1851	0.2303
	0.65	0.3561	0.1906	-0.2285	0.1846	-0.1642	0.1687	0.70	0.3414	0.2060	-0.1913	0.1904	-0.1426	0.1187
	0.75	0.3258	0.2193	-0.1548	0.1975	-0.1204	0.0789	0.80	0.3097	0.2304	-0.1196	0.2059	-0.0975	0.0483
	0.85	0.2935	0.2391	-0.0862	0.2154	-0.0739	0.0260	0.90	0.2779	0.2453	-0.0550	0.2261	-0.0496	0.0110
	0.95	0.2633	0.2489	-0.0263	0.2377	-0.0250	0.0026	1.00	0.2500	0.2500	0.0	0.2500	0.0	0.0

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

p	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)
0.55	0.05	0.4875	-0.0537	-0.2736	0.2263	-0.2761	2.8179	0.10	0.4691	-0.0425	-0.3821	0.1903	-0.3076	2.1869
	0.15	0.4616	-0.0231	-0.4368	0.1717	-0.3083	1.7443	0.20	0.4589	-0.0011	-0.4614	0.1600	-0.2986	1.4087
	0.25	0.4580	0.0216	-0.4665	0.1522	-0.2847	1.1440	0.30	0.4573	0.0444	-0.4583	0.1469	-0.2688	0.9295
	0.35	0.4557	0.0668	-0.4407	0.1433	-0.2519	0.7530	0.40	0.4528	0.0886	-0.4161	0.1412	-0.2348	0.6062
	0.45	0.4480	0.1097	-0.3864	0.1404	-0.2173	0.4836	0.50	0.4413	0.1300	-0.3531	0.1407	-0.1996	0.3808
	0.55	0.4326	0.1492	-0.3173	0.1423	-0.1817	0.2947	0.60	0.4220	0.1672	-0.2798	0.1444	-0.1635	0.2230
	0.65	0.4096	0.1839	-0.2416	0.1478	-0.1450	0.1638	0.70	0.3956	0.1990	-0.2032	0.1523	-0.1260	0.1155
	0.75	0.3804	0.2124	-0.1654	0.1580	-0.1065	0.0771	0.80	0.3645	0.2239	-0.1285	0.1647	-0.0864	0.0474
	0.85	0.3482	0.2333	-0.0932	0.1726	-0.0657	0.0256	0.90	0.3321	0.2404	-0.0599	0.1817	-0.0443	0.0109
	0.95	0.3168	0.2451	-0.0287	0.1917	-0.0224	0.0026	1.00	0.3025	0.2475	0.0	0.2025	0.0	0.0
0.60	0.05	0.5356	-0.0506	-0.2894	0.1895	-0.2680	2.8388	0.10	0.5173	-0.0384	-0.4006	0.1574	-0.2903	2.1778
	0.15	0.5100	-0.0194	-0.4556	0.1407	-0.2857	1.7233	0.20	0.5076	0.0015	-0.4797	0.1302	-0.2730	1.3840
	0.25	0.5071	0.0229	-0.4842	0.1230	-0.2575	1.1188	0.30	0.5069	0.0441	-0.4755	0.1179	-0.2410	0.9061
	0.35	0.5060	0.0649	-0.4572	0.1145	-0.2245	0.7324	0.40	0.5037	0.0851	-0.4320	0.1123	-0.2080	0.5888
	0.45	0.4998	0.1047	-0.4019	0.1111	-0.1915	0.4693	0.50	0.4941	0.1235	-0.3680	0.1109	-0.1754	0.3695
	0.55	0.4864	0.1414	-0.3315	0.1116	-0.1592	0.2861	0.60	0.4768	0.1584	-0.2933	0.1134	-0.1431	0.2168
	0.65	0.4654	0.1742	-0.2542	0.1156	-0.1268	0.1595	0.70	0.4524	0.1888	-0.2148	0.1190	-0.1102	0.1127
	0.75	0.4380	0.2019	-0.1756	0.1234	-0.0932	0.0754	0.80	0.4227	0.2134	-0.1372	0.1287	-0.0757	0.0465
	0.85	0.4067	0.2231	-0.1001	0.1350	-0.0577	0.0252	0.90	0.3906	0.2309	-0.0646	0.1424	-0.0390	0.0108
	0.95	0.3749	0.2365	-0.0312	0.1508	-0.0198	0.0026	1.00	0.3600	0.2400	0.0	0.1600	0.0	0.0
0.65	0.05	0.5846	-0.0465	-0.3086	0.1552	-0.2601	2.9075	0.10	0.5668	-0.0336	-0.4226	0.1272	-0.2730	2.1984
	0.15	0.5600	-0.0153	-0.4774	0.1126	-0.2632	1.7227	0.20	0.5580	0.0043	-0.5006	0.1033	-0.2477	1.3731
	0.25	0.5580	0.0239	-0.5040	0.0968	-0.2308	1.1038	0.30	0.5584	0.0432	-0.4940	0.0922	-0.2141	0.8901
	0.35	0.5581	0.0621	-0.4747	0.0889	-0.1978	0.7172	0.40	0.5566	0.0805	-0.4487	0.0867	-0.1820	0.5753
	0.45	0.5536	0.0982	-0.4176	0.0854	-0.1669	0.4579	0.50	0.5488	0.1152	-0.3829	0.0849	-0.1522	0.3602
	0.55	0.5423	0.1316	-0.3457	0.0852	-0.1378	0.2789	0.60	0.5338	0.1471	-0.3066	0.0861	-0.1236	0.2114
	0.65	0.5237	0.1617	-0.2665	0.0880	-0.1093	0.1557	0.70	0.5118	0.1754	-0.2260	0.0902	-0.0950	0.1103
	0.75	0.4986	0.1878	-0.1855	0.0934	-0.0803	0.0739	0.80	0.4841	0.1990	-0.1456	0.0975	-0.0654	0.0458
	0.85	0.4689	0.2088	-0.1067	0.1024	-0.0499	0.0249	0.90	0.4533	0.2168	-0.0693	0.1082	-0.0339	0.0107
	0.95	0.4377	0.2231	-0.0336	0.1149	-0.0172	0.0026	1.00	0.4225	0.2275	0.0	0.1225	0.0	0.0
0.70	0.05	0.6325	-0.0414	-0.3325	0.1235	-0.2519	3.0322	0.10	0.6179	-0.0281	-0.4491	0.0997	-0.2549	2.2510
	0.15	0.6118	-0.0109	-0.5031	0.0872	-0.2400	1.7420	0.20	0.6105	0.0069	-0.5245	0.0791	-0.2219	1.3756
	0.25	0.6110	0.0244	-0.5260	0.0735	-0.2042	1.0981	0.30	0.6119	0.0417	-0.5144	0.0695	-0.1871	0.8807
	0.35	0.6122	0.0583	-0.4935	0.0708	-0.1714	0.7067	0.40	0.6115	0.0744	-0.4661	0.0645	-0.1567	0.5651
	0.45	0.6094	0.0900	-0.4338	0.0632	-0.1429	0.4488	0.50	0.6057	0.1051	-0.3981	0.0626	-0.1298	0.3526
	0.55	0.6002	0.1196	-0.3598	0.0625	-0.1171	0.2728	0.60	0.5931	0.1334	-0.3198	0.0630	-0.1047	0.2068
	0.65	0.5843	0.1466	-0.2787	0.0641	-0.0925	0.1524	0.70	0.5738	0.1589	-0.2370	0.0657	-0.0803	0.1080
	0.75	0.5619	0.1704	-0.1952	0.0680	-0.0680	0.0726	0.80	0.5488	0.1809	-0.1539	0.0709	-0.0553	0.0450
	0.85	0.5347	0.1903	-0.1133	0.0745	-0.0423	0.0246	0.90	0.5199	0.1984	-0.0739	0.0789	-0.0288	0.0106
	0.95	0.5049	0.2050	-0.0360	0.0841	-0.0147	0.0026	1.00	0.4900	0.2100	0.0	0.0900	0.0	0.0
0.75	0.05	0.6875	-0.0354	-0.3629	0.0943	-0.2427	3.2281	0.10	0.6709	-0.0222	-0.4814	0.0747	-0.2353	2.3414
	0.15	0.6659	-0.0065	-0.5335	0.0644	-0.2153	1.7830	0.20	0.6653	0.0091	-0.5522	0.0577	-0.1953	1.3919
	0.25	0.6664	0.0244	-0.5510	0.0531	-0.1766	1.1014	0.30	0.6678	0.0391	-0.5368	0.0497	-0.1601	0.8775
	0.35	0.6687	0.0533	-0.5138	0.0472	-0.1452	0.7005	0.40	0.6687	0.0670	-0.4845	0.0455	-0.1318	0.5580
	0.45	0.6674	0.0802	-0.4507	0.0443	-0.1194	0.4419	0.50	0.6647	0.0931	-0.4136	0.0437	-0.1079	0.3465
	0.55	0.6604	0.1054	-0.3741	0.0434	-0.0970	0.2677	0.60	0.6546	0.1173	-0.3329	0.0436	-0.0865	0.2028
	0.65	0.6472	0.1287	-0.2906	0.0443	-0.0763	0.1495	0.70	0.6384	0.1396	-0.2477	0.0454	-0.0661	0.1061
	0.75	0.6281	0.1498	-0.2047	0.0468	-0.0559	0.0714	0.80	0.6165	0.1593	-0.1619	0.0488	-0.0456	0.0444
	0.85	0.6039	0.1679	-0.1197	0.0513	-0.0349	0.0243	0.90	0.5905	0.1756	-0.0784	0.0544	-0.0238	0.0105
	0.95	0.5766	0.1822	-0.0384	0.0581	-0.0122	0.0026	1.00	0.5625	0.1875	0.0	0.0625	0.0	0.0

SCALED ELEMENTS OF INFORMATION MATRIX FOR A MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

p	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)	h	J(1,1)	J(1,2)	J(1,3)	J(2,2)	J(2,3)	J(3,3)
0.80	0.05 *	0.7406	-0.0285	-0.4026	0.0678	-0.2312	3.5252	0.10 *	0.7264	-0.0158	-0.5218	0.0524	-0.2124	2.4786
	0.15 *	0.7228	-0.0023	-0.5702	0.0443	-0.1882	1.8495	0.20 *	0.7230	0.0109	-0.5846	0.0391	-0.1663	1.4230
	0.25 *	0.7247	0.0234	-0.5793	0.0355	-0.1479	1.1140	0.30 *	0.7265	0.0353	-0.5617	0.0329	-0.1322	0.8804
	0.35 *	0.7279	0.0468	-0.5358	0.0310	-0.1187	0.6985	0.40 *	0.7285	0.0579	-0.5041	0.0297	-0.1068	0.5537
	0.45 *	0.7279	0.0686	-0.4683	0.0287	-0.0961	0.4369	0.50 *	0.7261	0.0790	-0.4295	0.0281	-0.0864	0.3416
	0.55 *	0.7230	0.0891	-0.3885	0.0279	-0.0773	0.2635	0.60 *	0.7185	0.0989	-0.3460	0.0279	-0.0687	0.1994
	0.65 *	0.7127	0.1083	-0.3025	0.0282	-0.0604	0.1469	0.70 *	0.7055	0.1173	-0.2584	0.0289	-0.0523	0.1043
	0.75 *	0.6970	0.1259	-0.2140	0.0297	-0.0442	0.0702	0.80 *	0.6873	0.1341	-0.1698	0.0310	-0.0360	0.0437
	0.85 *	0.6766	0.1416	-0.1259	0.0326	-0.0276	0.0240	0.90 *	0.6650	0.1486	-0.0828	0.0346	-0.0189	0.0104
	0.95 *	0.6527	0.1547	-0.0408	0.0371	-0.0097	0.0026	1.00 *	0.6400	0.1600	0.0	0.0400	0.0	0.0
0.85	0.05 *	0.7961	-0.0208	-0.4570	0.0442	-0.2152	3.9817	0.10 *	0.7855	-0.0095	-0.5739	0.0330	-0.1846	2.6812
	0.15 *	0.7831	0.0015	-0.6154	0.0272	-0.1563	1.9480	0.20 *	0.7841	0.0116	-0.6230	0.0236	-0.1344	1.4715
	0.25 *	0.7862	0.0210	-0.6120	0.0211	-0.1171	1.1368	0.30 *	0.7884	0.0300	-0.5897	0.0193	-0.1031	0.8896
	0.35 *	0.7901	0.0386	-0.5600	0.0180	-0.0914	0.7004	0.40 *	0.7910	0.0469	-0.5252	0.0171	-0.0815	0.5521
	0.45 *	0.7911	0.0550	-0.4868	0.0164	-0.0728	0.4337	0.50 *	0.7901	0.0629	-0.4460	0.0160	-0.0650	0.3380
	0.55 *	0.7881	0.0705	-0.4032	0.0157	-0.0579	0.2601	0.60 *	0.7849	0.0780	-0.3592	0.0157	-0.0513	0.1965
	0.65 *	0.7806	0.0852	-0.3143	0.0158	-0.0450	0.1447	0.70 *	0.7752	0.0922	-0.2689	0.0161	-0.0389	0.1027
	0.75 *	0.7686	0.0990	-0.2232	0.0166	-0.0328	0.0692	0.80 *	0.7611	0.1055	-0.1775	0.0173	-0.0268	0.0432
	0.85 *	0.7526	0.1117	-0.1321	0.0182	-0.0205	0.0237	0.90 *	0.7432	0.1174	-0.0872	0.0193	-0.0141	0.0103
	0.95 *	0.7331	0.1227	-0.0431	0.0208	-0.0073	0.0025	1.00 *	0.7225	0.1275	0.0	0.0225	0.0	0.0
0.90	0.05 *	0.8551	-0.0124	-0.5370	0.0239	-0.1890	4.7196	0.10 *	0.8482	-0.0036	-0.6437	0.0169	-0.1468	2.9832
	0.15 *	0.8480	0.0040	-0.6724	0.0134	-0.1179	2.0898	0.20 *	0.8497	0.0107	-0.6694	0.0114	-0.0979	1.5419
	0.25 *	0.8519	0.0169	-0.6501	0.0100	-0.0832	1.1715	0.30 *	0.8540	0.0228	-0.6214	0.0090	-0.0719	0.9055
	0.35 *	0.8557	0.0284	-0.5866	0.0083	-0.0629	0.7064	0.40 *	0.8569	0.0339	-0.5480	0.0078	-0.0555	0.5530
	0.45 *	0.8573	0.0392	-0.5065	0.0074	-0.0491	0.4321	0.50 *	0.8569	0.0444	-0.4631	0.0072	-0.0436	0.3353
	0.55 *	0.8558	0.0496	-0.4183	0.0070	-0.0386	0.2574	0.60 *	0.8538	0.0546	-0.3726	0.0070	-0.0341	0.1941
	0.65 *	0.8510	0.0595	-0.3261	0.0070	-0.0298	0.1427	0.70 *	0.8474	0.0643	-0.2793	0.0071	-0.0257	0.1013
	0.75 *	0.8430	0.0690	-0.2322	0.0073	-0.0217	0.0683	0.80 *	0.8378	0.0736	-0.1851	0.0076	-0.0177	0.0426
	0.85 *	0.8318	0.0780	-0.1382	0.0080	-0.0136	0.0235	0.90 *	0.8252	0.0822	-0.0915	0.0085	-0.0093	0.0103
	0.95 *	0.8179	0.0863	-0.0454	0.0092	-0.0048	0.0025	1.00 *	0.8100	0.0900	0.0	0.0100	0.0	0.0
0.95	0.05 *	0.9205	-0.0041	-0.6695	0.0079	-0.1378	6.0630	0.10 *	0.9182	0.0006	-0.7431	0.0051	-0.0920	3.4541
	0.15 *	0.9192	0.0042	-0.7470	0.0038	-0.0686	2.2962	0.20 *	0.9209	0.0073	-0.7268	0.0031	-0.0544	1.6404
	0.25 *	0.9227	0.0102	-0.6953	0.0027	-0.0449	1.2199	0.30 *	0.9242	0.0130	-0.6577	0.0024	-0.0379	0.9289
	0.35 *	0.9255	0.0157	-0.6165	0.0022	-0.0326	0.7165	0.40 *	0.9263	0.0184	-0.5728	0.0020	-0.0284	0.5563
	0.45 *	0.9268	0.0210	-0.5275	0.0019	-0.0250	0.4320	0.50 *	0.9268	0.0236	-0.4811	0.0018	-0.0220	0.3338
	0.55 *	0.9264	0.0261	-0.4339	0.0018	-0.0194	0.2553	0.60 *	0.9255	0.0286	-0.3861	0.0018	-0.0170	0.1921
	0.65 *	0.9242	0.0311	-0.3380	0.0018	-0.0148	0.1410	0.70 *	0.9224	0.0336	-0.2896	0.0018	-0.0128	0.1000
	0.75 *	0.9201	0.0360	-0.2411	0.0018	-0.0107	0.0674	0.80 *	0.9174	0.0384	-0.1926	0.0019	-0.0088	0.0421
	0.85 *	0.9143	0.0408	-0.1441	0.0020	-0.0067	0.0232	0.90 *	0.9108	0.0431	-0.0958	0.0021	-0.0046	0.0102
	0.95 *	0.9068	0.0453	-0.0477	0.0023	-0.0024	0.0025	1.00 *	0.9025	0.0475	0.0	0.0025	0.0	0.0

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